CS134:
Sorting

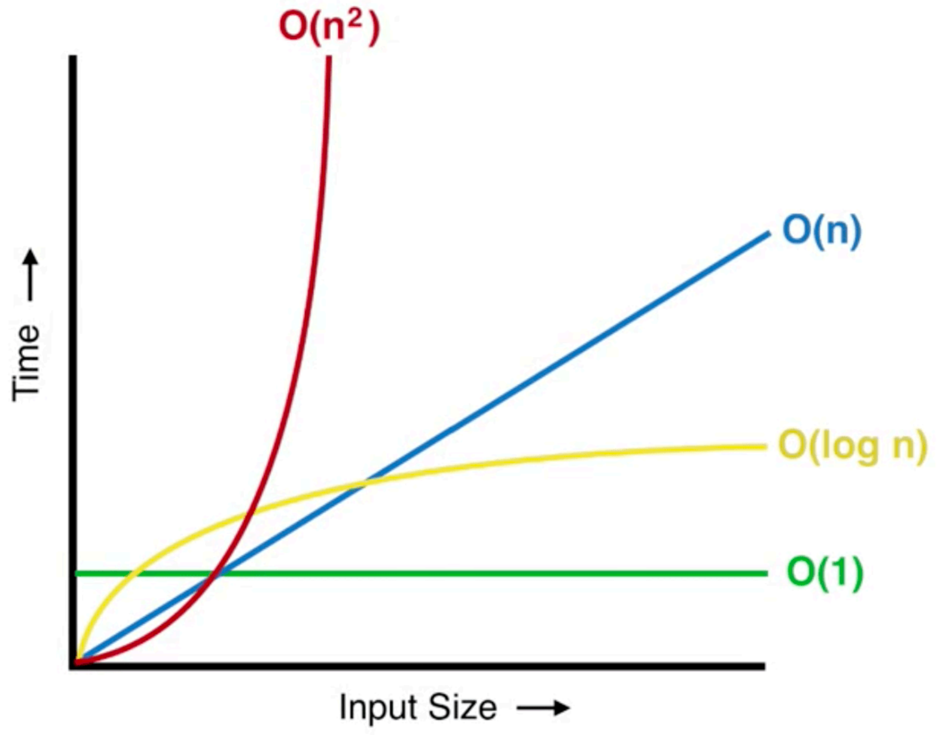
Announcements & Logistics

- **Lab 9 Boggle**
  - Work on Boggle again in lab this week today/tomorrow
  - **All three parts** are due Wed/Thur at 11 pm
- **HW 9** will be released on Wed, due next Mon @ 11 pm
- Check calendar for updated office hours this week
- Last lab (**Lab 10**) will be a short Java program
- We will discuss Java in last few lectures after we wrap up sorting today

Do You Have Any Questions?

Last Time: Efficiency & Searching

- Measured efficiency as number of steps taken by algorithm on worst-case inputs of a given size
- Introduced Big-O notation which captures the rate at which the number of steps taken by the algorithm grows wrt size of input n, "as n gets large"
- Compared array lists vs linked lists
- Compared linear vs binary search

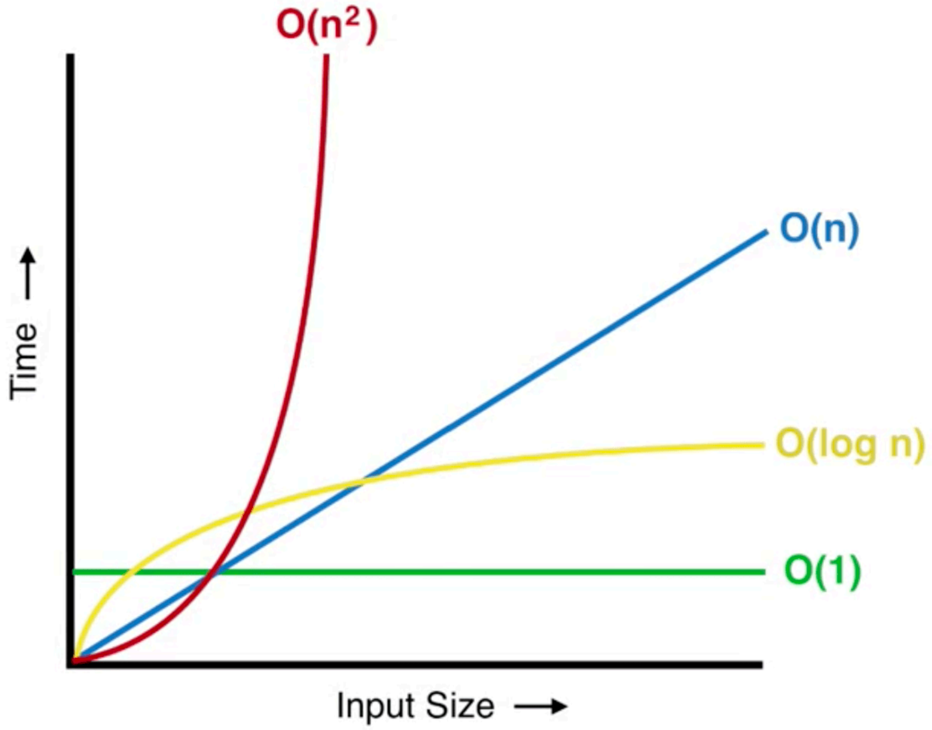


Today: Searching and Sorting

- Wrap up our discussion of binary search including a runtime analysis
- Discuss some classic sorting algorithms:
    - ***Selection sorting*** in $O(n^2)$ time
    - A brief (high level) discussion of how we can improve it to $O(n \log n)$
    - Overview of recursive ***merge sort*** algorithm

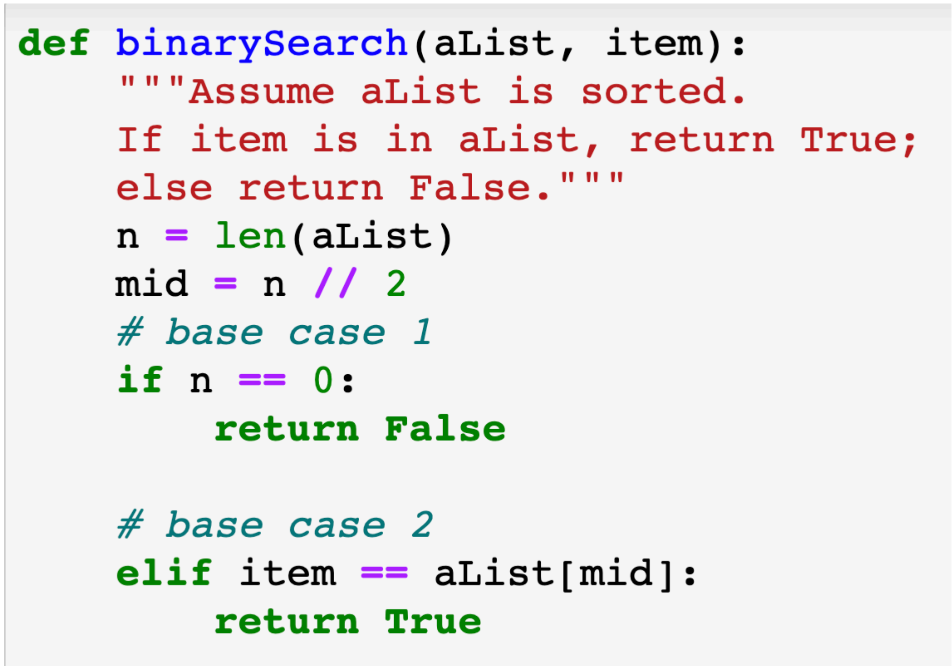
Review: Binary Search

- **Binary search:** recursive search algorithm to search in a **sorted array list**
    - Similar to how we search for a word in a (physical) dictionary
    - Takes $O(\log n)$ time
- Much more efficient than a **linear search**
- **Note:** $\log n$ grows much more slowly compared to n as n gets large



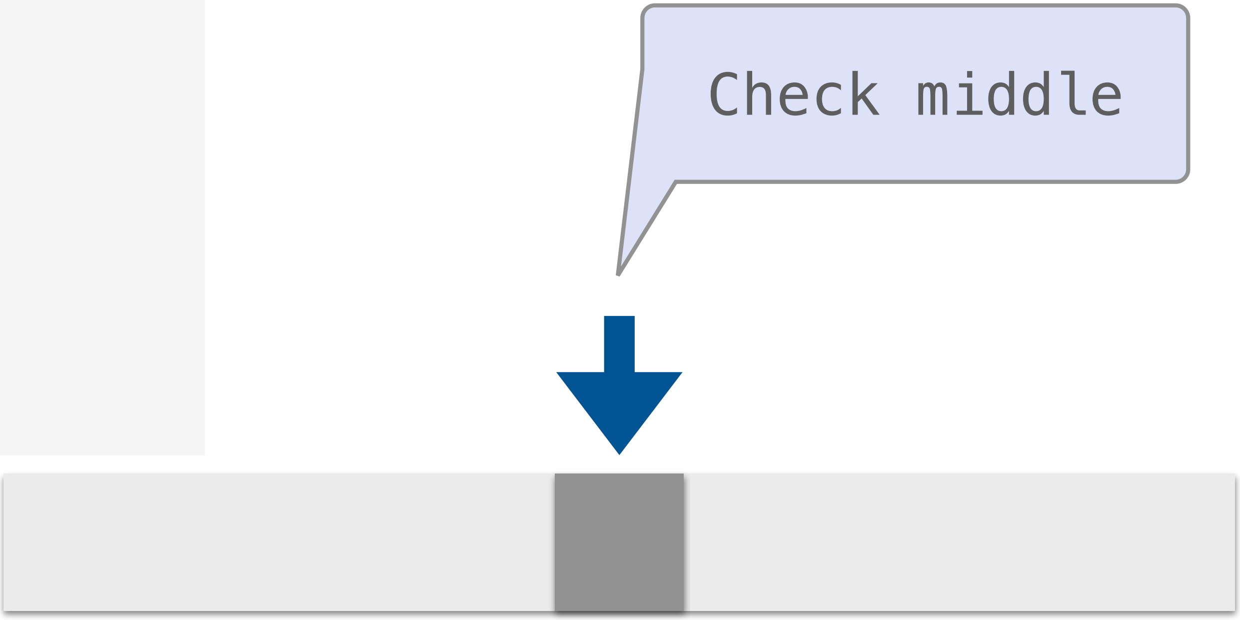
Review: Binary Search

- Base cases? When are we done?
    - If list is too small (or empty) to continue searching
    - If item we're searching for is the middle element



```python
def binarySearch(aList, item):
    """Assume aList is sorted.
    If item is in aList, return True;
    else return False."""
    n = len(aList)
    mid = n // 2
    # base case 1
    if n == 0:
        return False

    # base case 2
    elif item == aList[mid]:
        return True
```



Check middle

mid = n//2

Review: Binary Search

- Recursive case:
    - Recurse on left side if item is smaller than middle
    - Recurse on right side if item is larger than middle

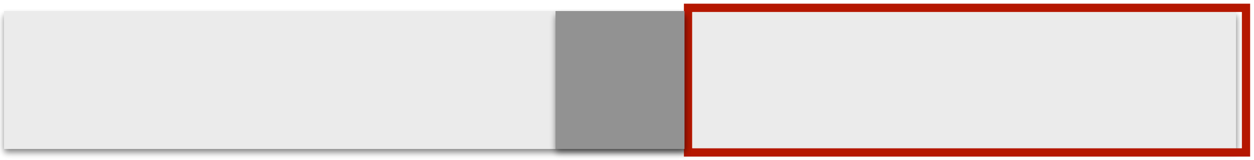
If item < L[mid], then need to search in L[:mid]

`mid = n//2`

Review: Binary Search

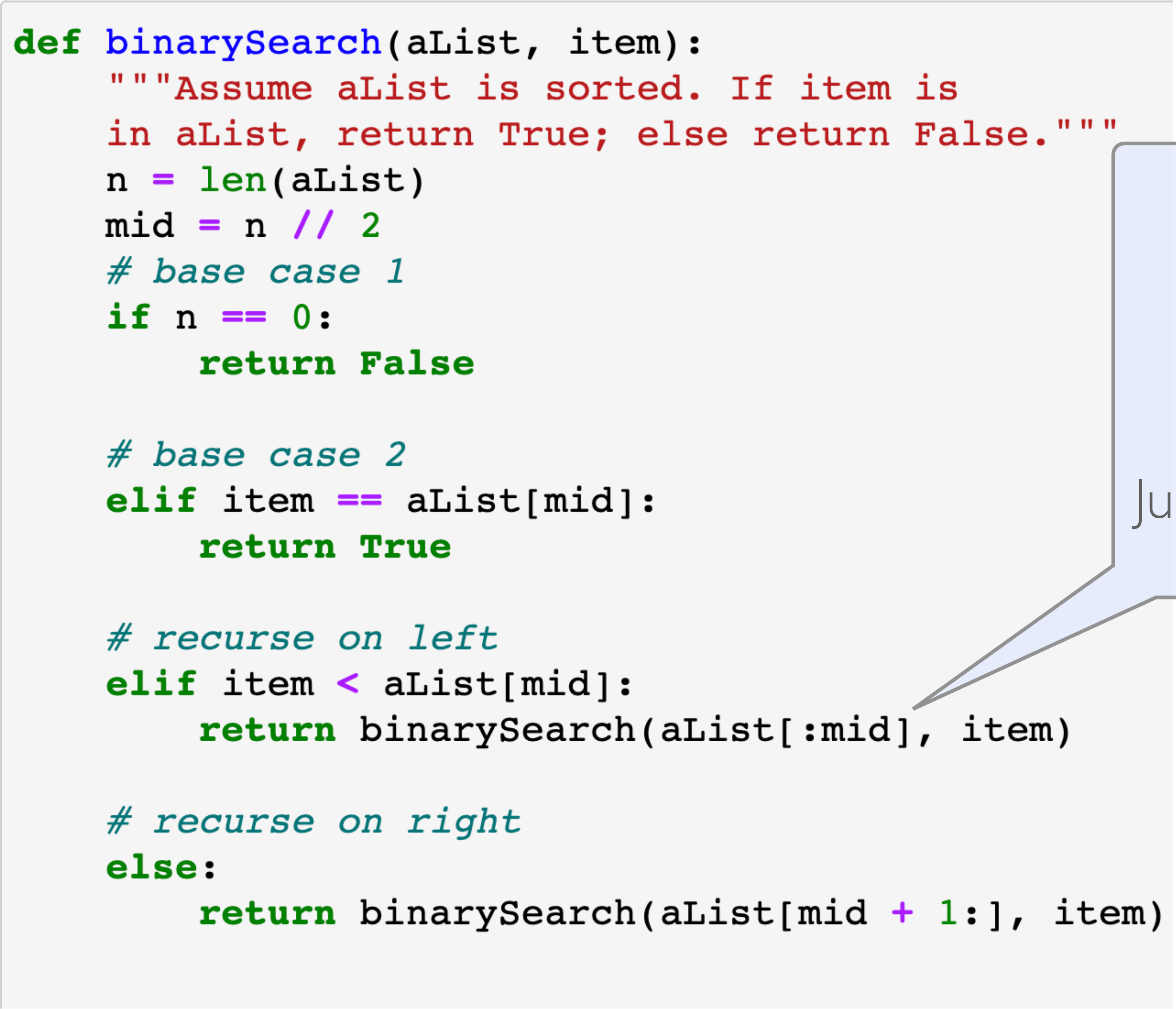
- Recursive case:
  - Recurse on left side if item is smaller than middle
  - Recurse on right side if item is larger than middle

If item > L[mid], then need to search in L[mid+1:]



`mid = n//2`

Review: Binary Search



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    """Assume aList is sorted. If item is
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    n = len(aList)
    mid = n // 2
    # base case 1
    if n == 0:
        return False

    # base case 2
    elif item == aList[mid]:
        return True

    # recurse on left
    elif item < aList[mid]:
        return binarySearch(aList[:mid], item)

    # recurse on right
    else:
        return binarySearch(aList[mid + 1:], item)
```

There is one small problem with our implementation. List splicing is O(n)! See Jupyter for improvement.

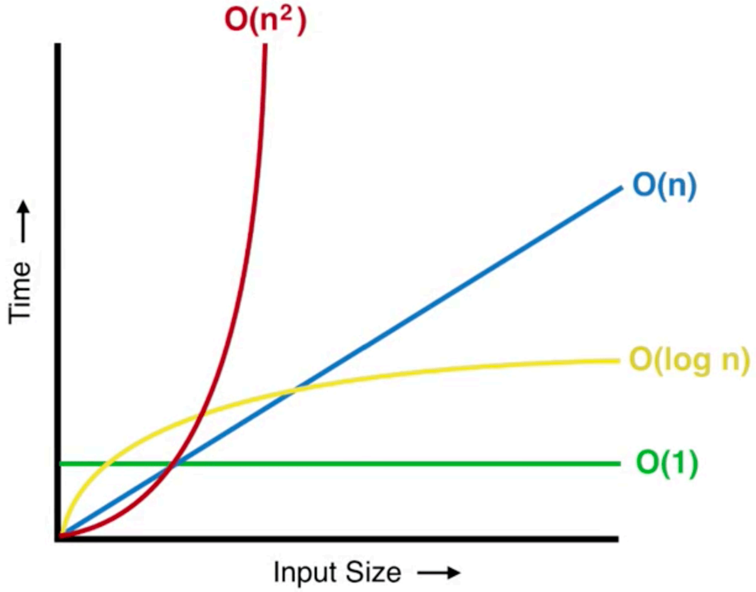
Analysis of Binary Search

- Within a recursive call in our improved approach:
  - Constant number of steps (independent of n): just 1 comparison
  - Therefore total number of steps: $O(\#$ of recursive calls)
- Size of list gets cut in half in each recursive call:

$$n \rightarrow n/2 \rightarrow n/4 \rightarrow n/8 \rightarrow \cdots \rightarrow n/2^i = 1$$

- This is an $O(\log n)$ time
- Really small even for large n!

$\log_2$ (1 billion) ~ 30



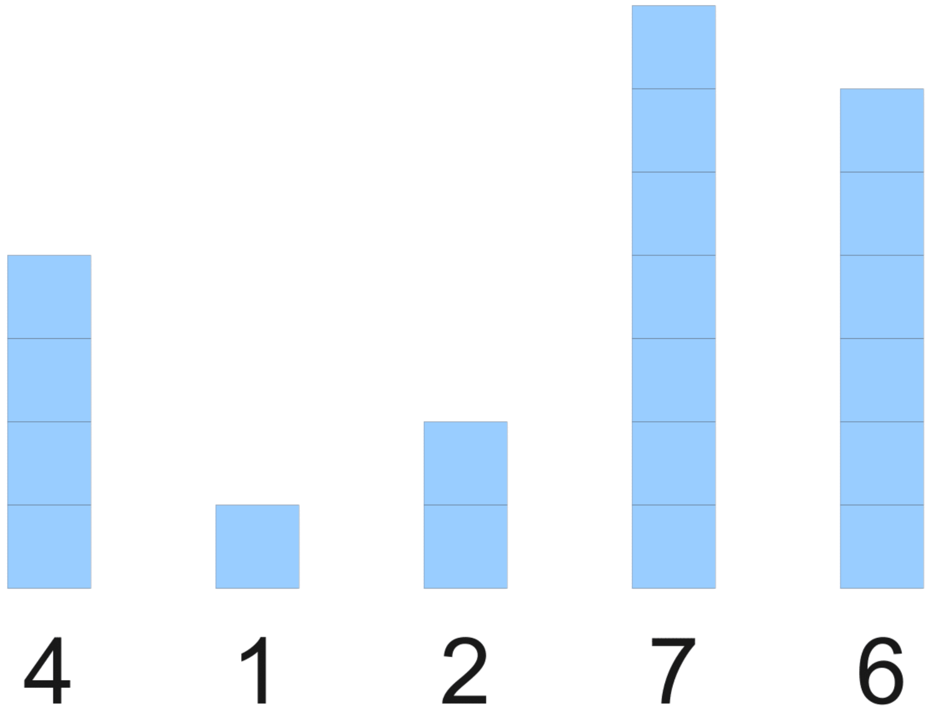
Sorting

Sorting

- **Problem:** Given a sequence of unordered elements, we need to sort the elements in ascending order.
- There are many ways to solve this problem!
- Built-in sorting functions/methods in Python
  - `sorted()`: function that returns a new sorted list
  - `sort()`: method that mutates and sorts the list its called on
- **Today:** how do we design our own sorting algorithm?
- **Question:** What is the best (most efficient) way to sort n items?
- We will use Big-O to find out!

Selection Sort

- A possible approach to sorting elements in a list/array:
  - Find the smallest element and move (swap) it to the first position
  - Repeat: find the second-smallest element and move it to the second position, and so on



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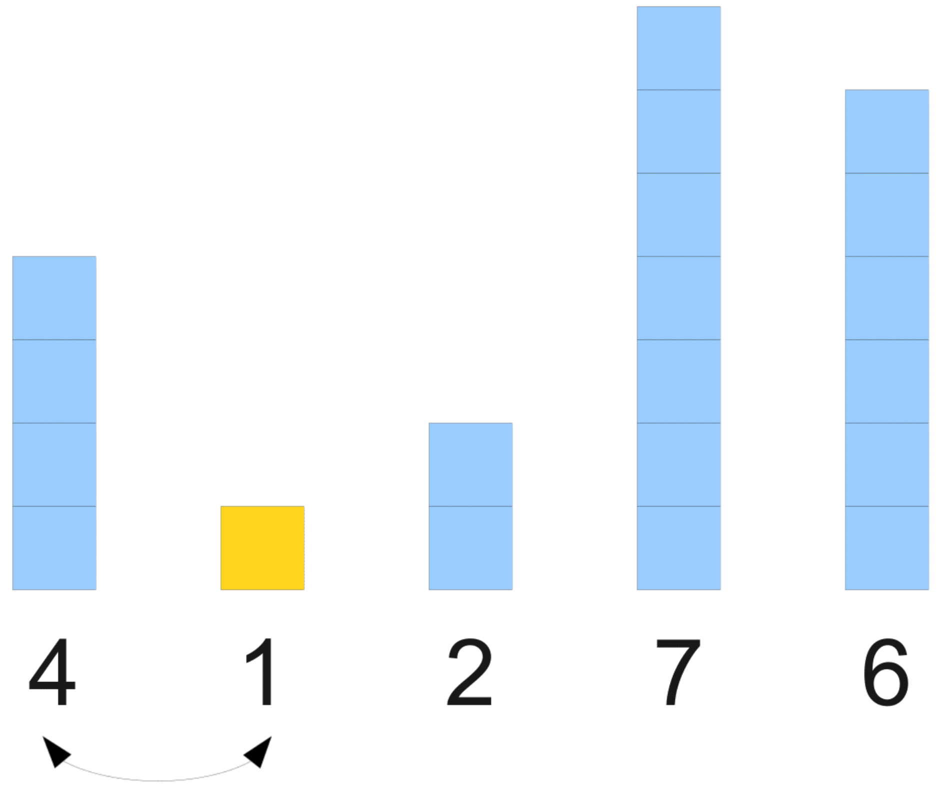

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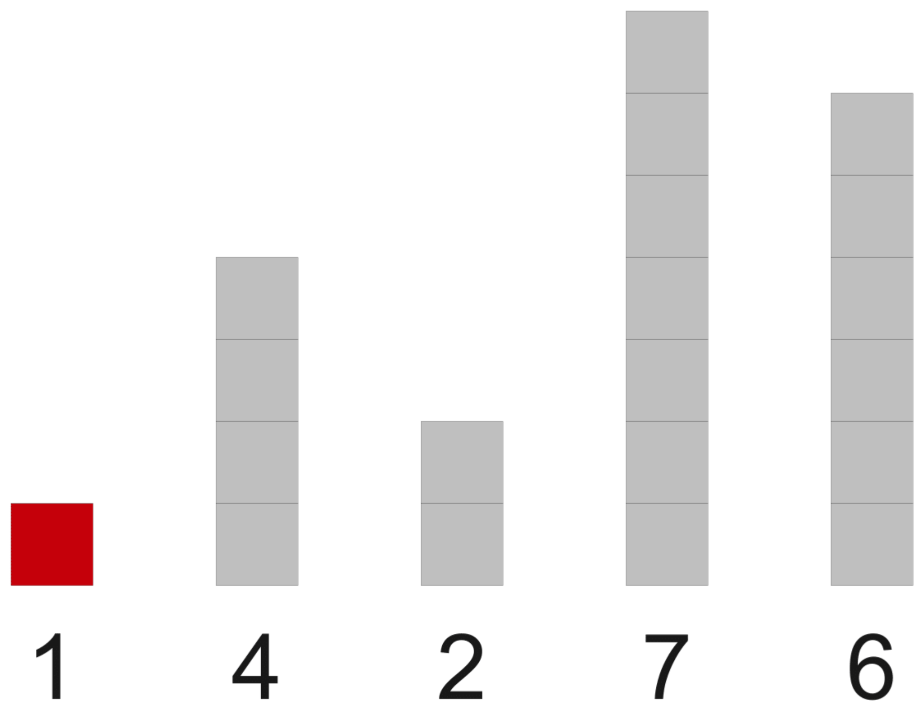

Selection Sort

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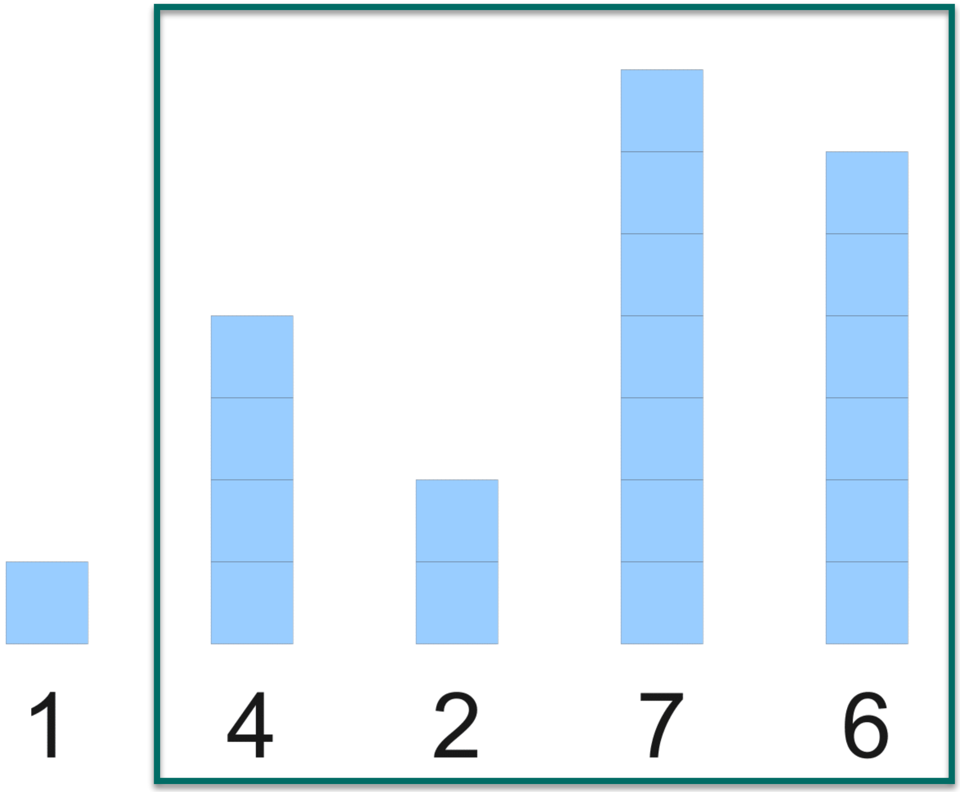

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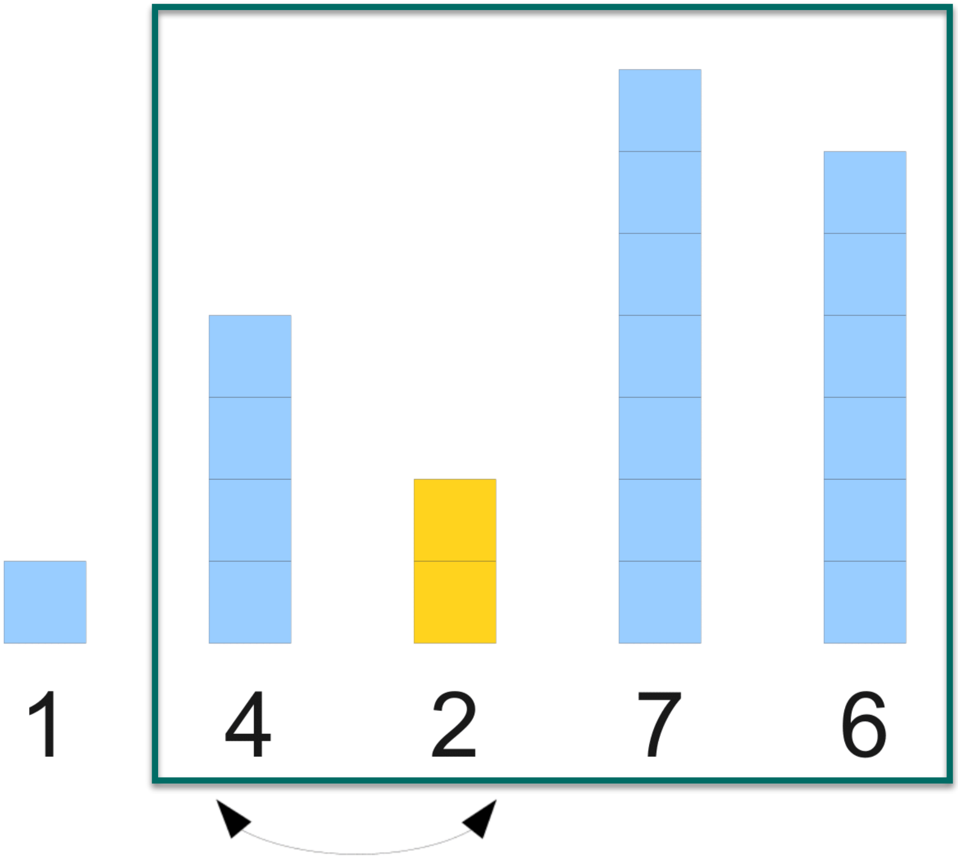

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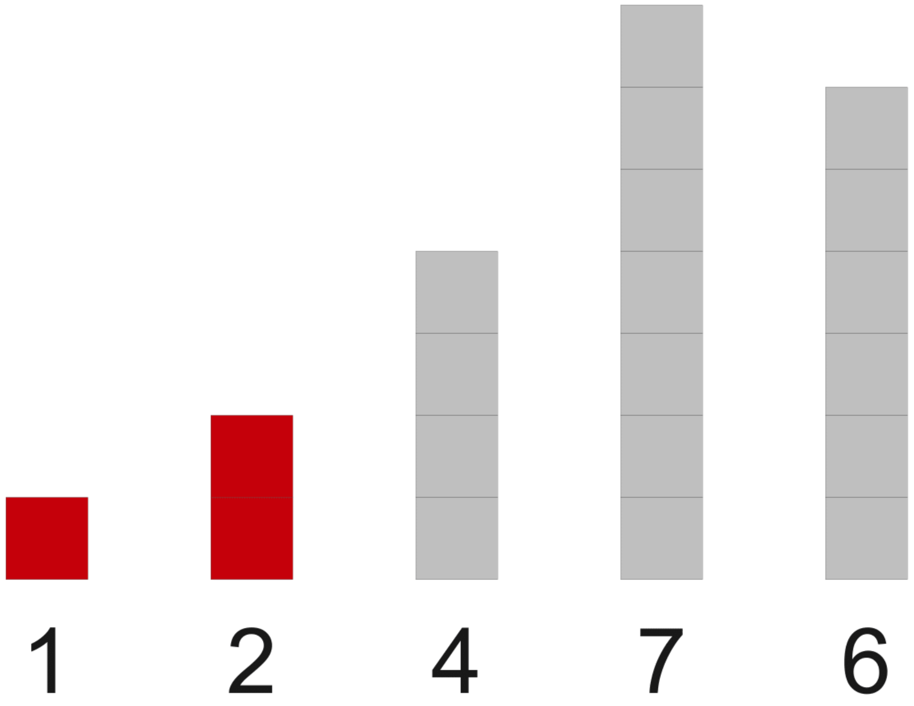

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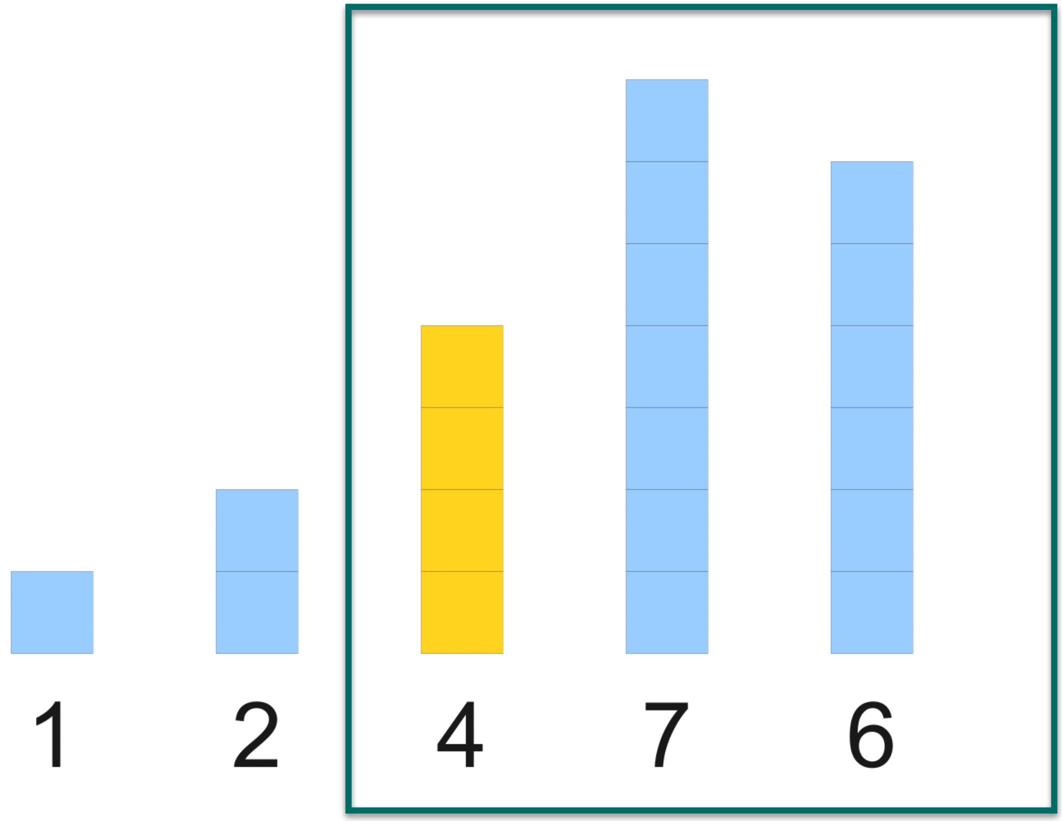

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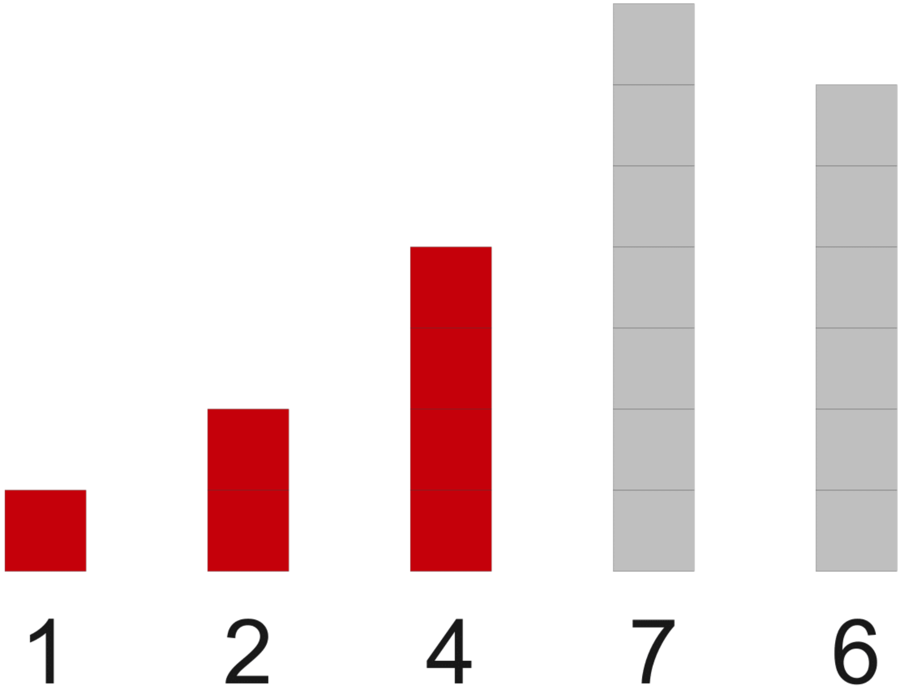

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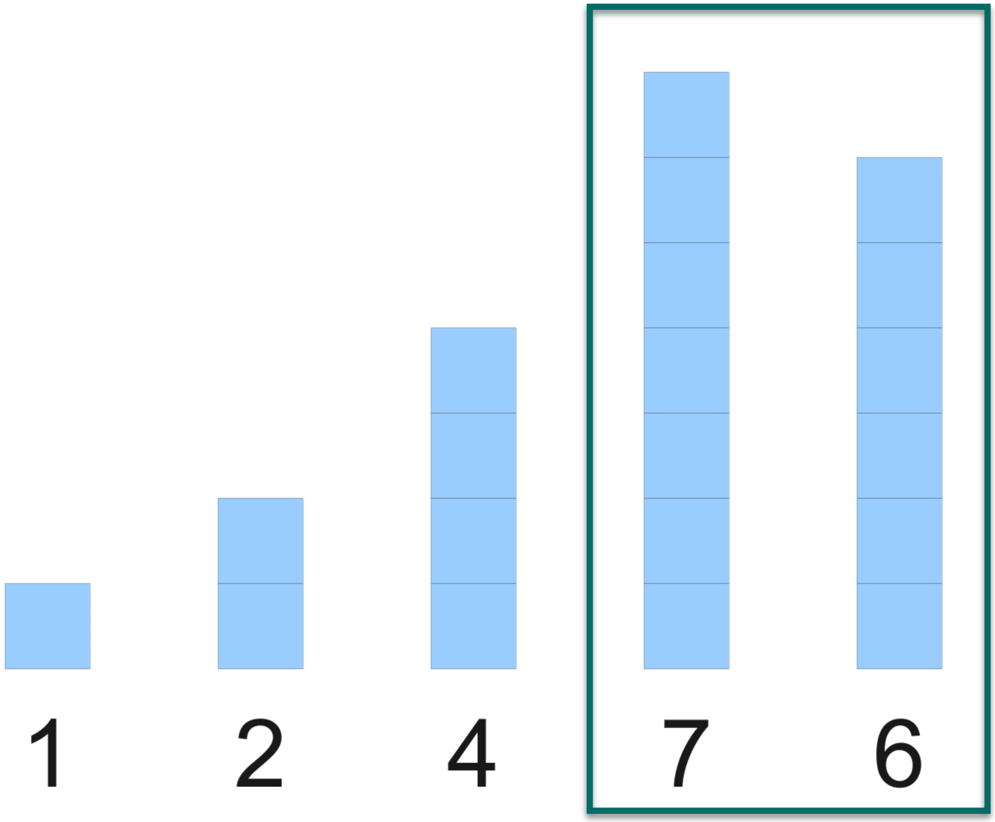

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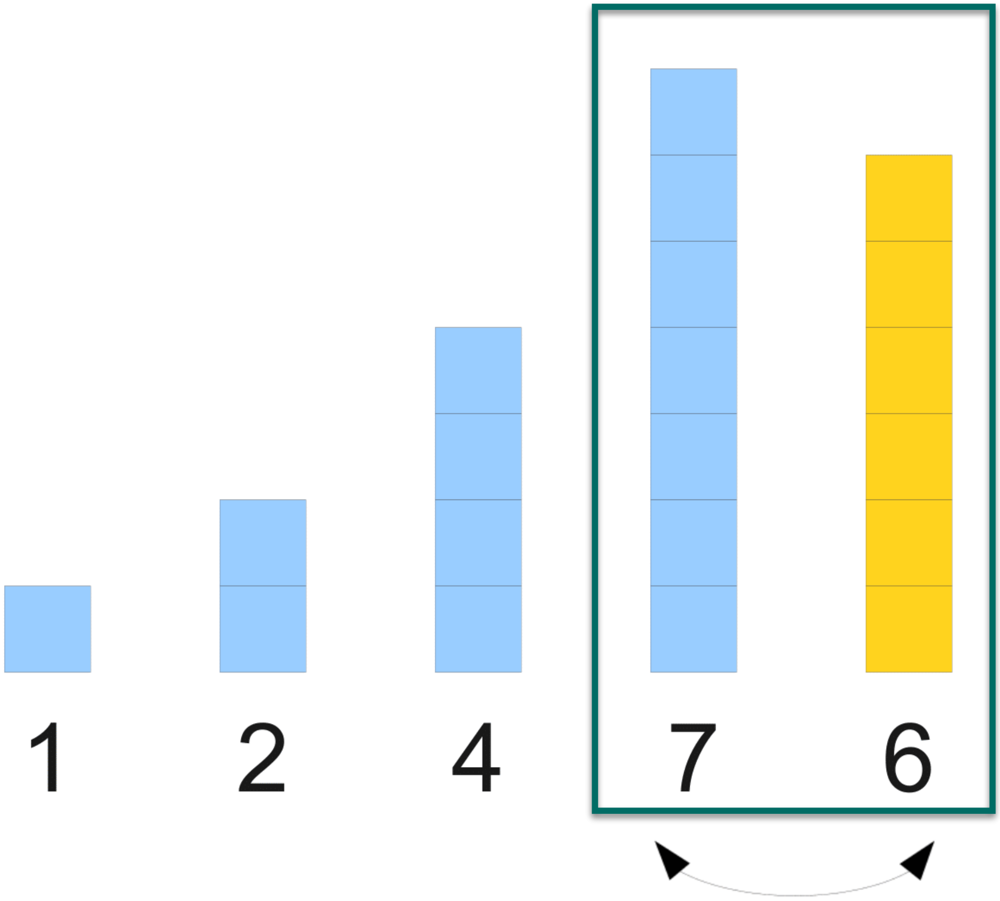
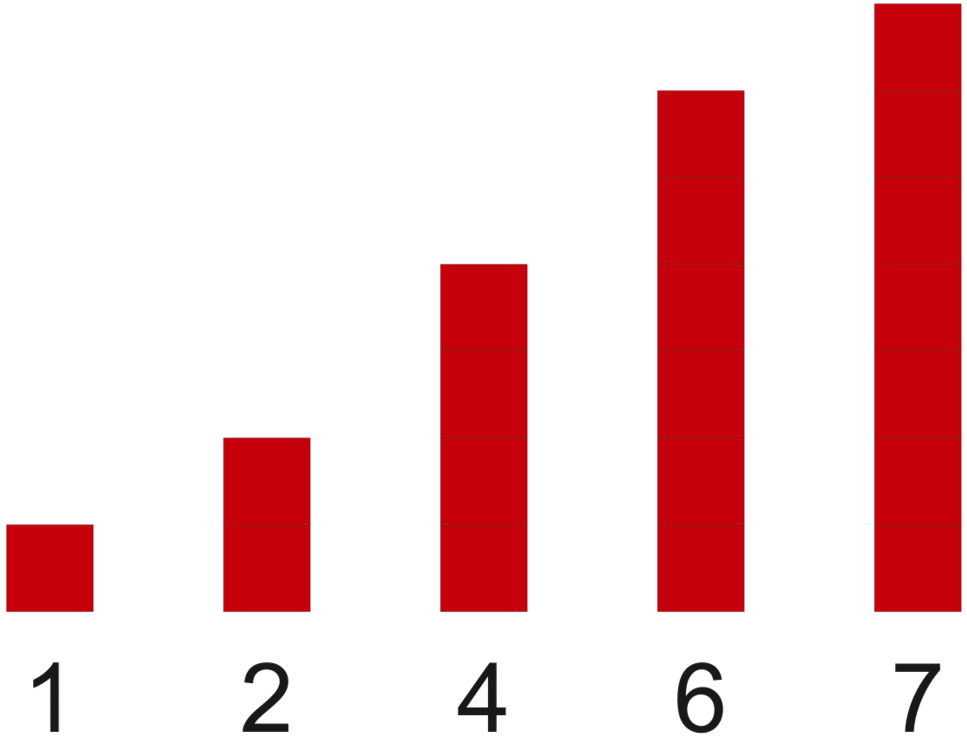


Selection Sort

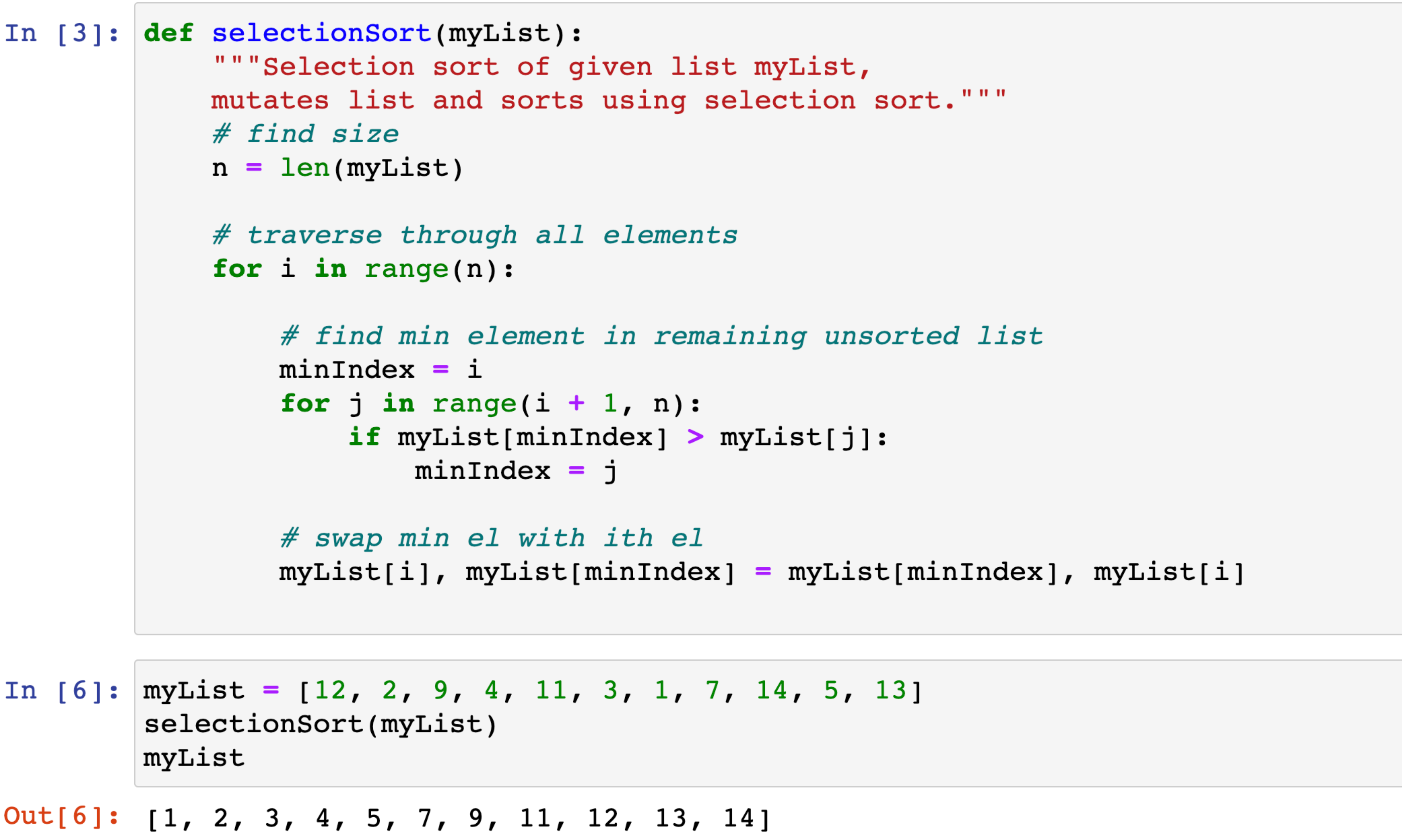
- Find the smallest element and move it to the first position and repeat



Selection Sort

- Generalize: For each index *i* in the list `L`, we need to find the **min** item in `L[i:]` so we can replace `L[i]` with that item
- In fact we need to find the position `minIndex` of the item that is minimum in `L[i:]`
- **Reminder:** how to swap values of variables `a` and `b`?
  - Using tuple assignment in Python: `a, b = b, a`
  - Or using a temp variable: `temp = a; a = b; b = temp`
- Let's implement this algorithm!

Selection Sort Code



```
In [3]: def selectionSort(myList):
            """Selection sort of given list myList,
            mutates list and sorts using selection sort."""
            # find size
            n = len(myList)

            # traverse through all elements
            for i in range(n):

                # find min element in remaining unsorted list
                minIndex = i
                for j in range(i + 1, n):
                    if myList[minIndex] > myList[j]:
                        minIndex = j

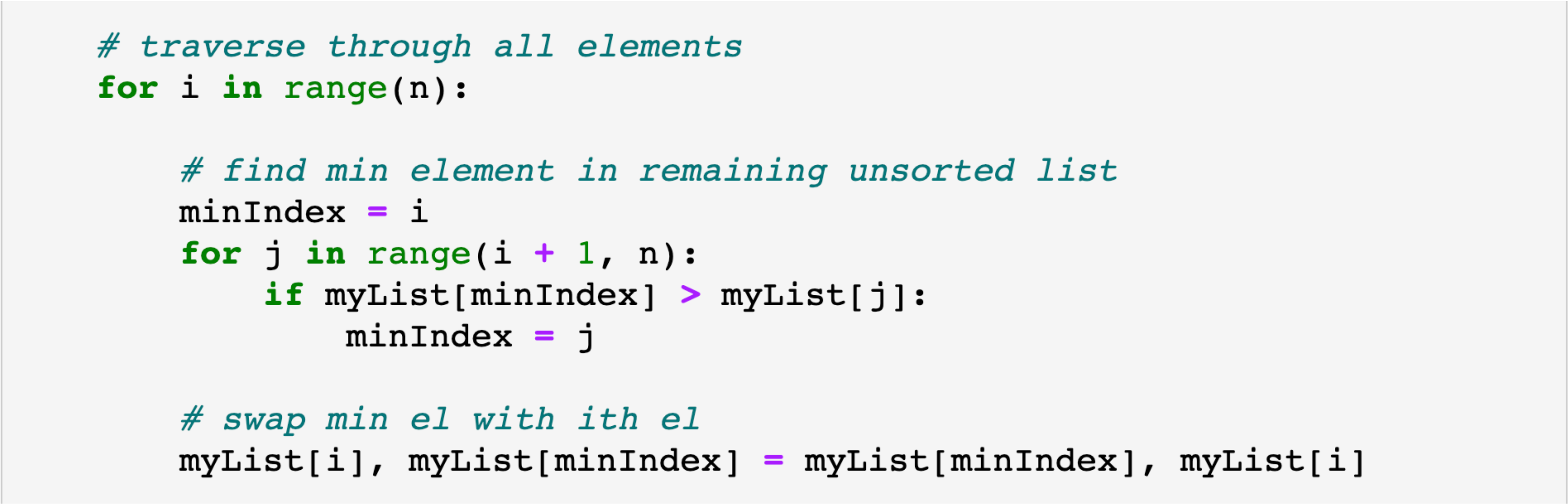
                # swap min el with ith el
                myList[i], myList[minIndex] = myList[minIndex], myList[i]
```

```
In [6]: myList = [12, 2, 9, 4, 11, 3, 1, 7, 14, 5, 13]
        selectionSort(myList)
        myList

Out[6]: [1, 2, 3, 4, 5, 7, 9, 11, 12, 13, 14]
```

Selection Sort Analysis

- For $i = 0$, inner loop runs $n - 1$ items
- For $i = 1$, inner loop runs $n - 2$ times
- ...
- For $i = n - 1$, inner loop runs 0 times



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# traverse through all elements
for i in range(n):

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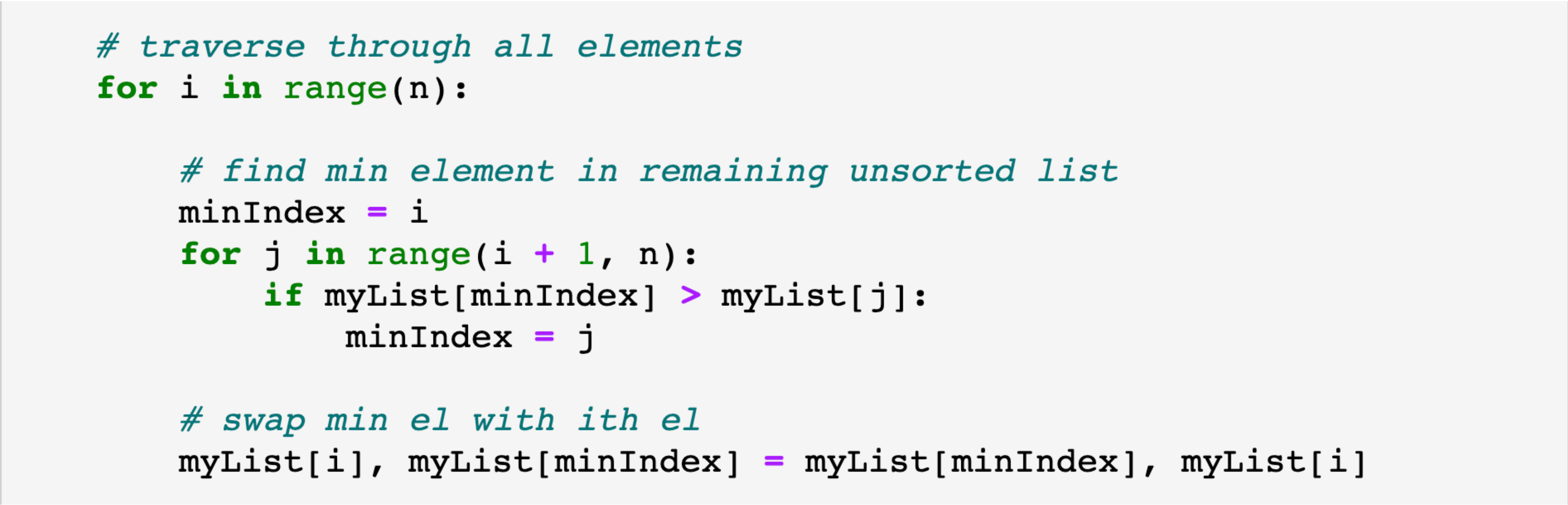
            minIndex = j

    # swap min el with ith el
    myList[i], myList[minIndex] = myList[minIndex], myList[i]
```

Selection Sort Analysis

- Within the inner loop we have $O(1)$ steps - just 1 comparison (constant)
- Thus overall number of steps is sum of inner loop steps $(n-1)+(n-2)+\cdots+0 \leq n+(n-1)+(n-2)+\cdots+1$
- What is this sum? (Math 200??)



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```

Selection Sort Analysis

$$
\begin{array}{rl}
 & S = n + (n-1) + (n-2) + \cdots + 2 + 1 \\
+ & S = 1 + 2 + \cdots + (n-2) + (n-1) + n \\
\hline
\end{array}
$$

$$2S = (n+1) + (n+1) + \cdots + (n+1) + (n+1) + (n+1)$$

$$2S = (n+1) \cdot n$$

$$S = (n+1) \cdot n \cdot 1/2$$

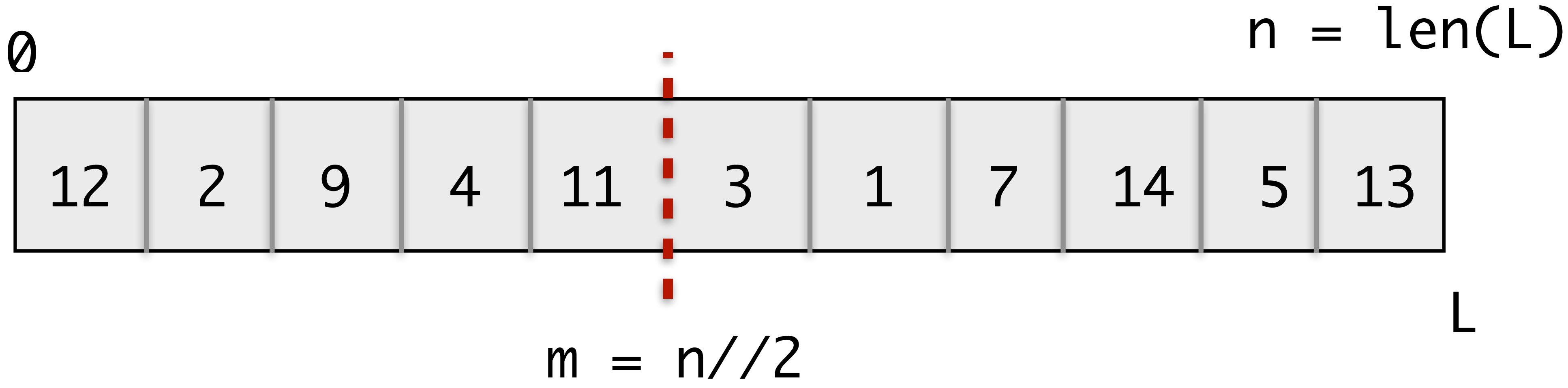
- Total number of steps taken by selection sort is thus:
  - $O(n(n+1)/2) \ = O(n(n+1)) \ = O(n^2+n) \ = O(n^2)$

Towards an $O(n \log n)$ Algorithm

- There are many other natural sorting algorithms that compare and rearrange elements in a slightly different way, but they are still $O(n^2)$ steps
  - Any algorithm that takes k steps to move each item k positions to its final position will take at least $O(n^2)$ steps as every element can be $O(n)$ away from its position in the worst case.
  - To do better than much better than n^2, we need to be able to move an item to its final position in significantly less steps
- Turns out we can sort in $O(n \log n)$ time if we are bit more clever, which is the best possible: ***Merge sort algorithm*** (Invented by John von Neumann in 1945)

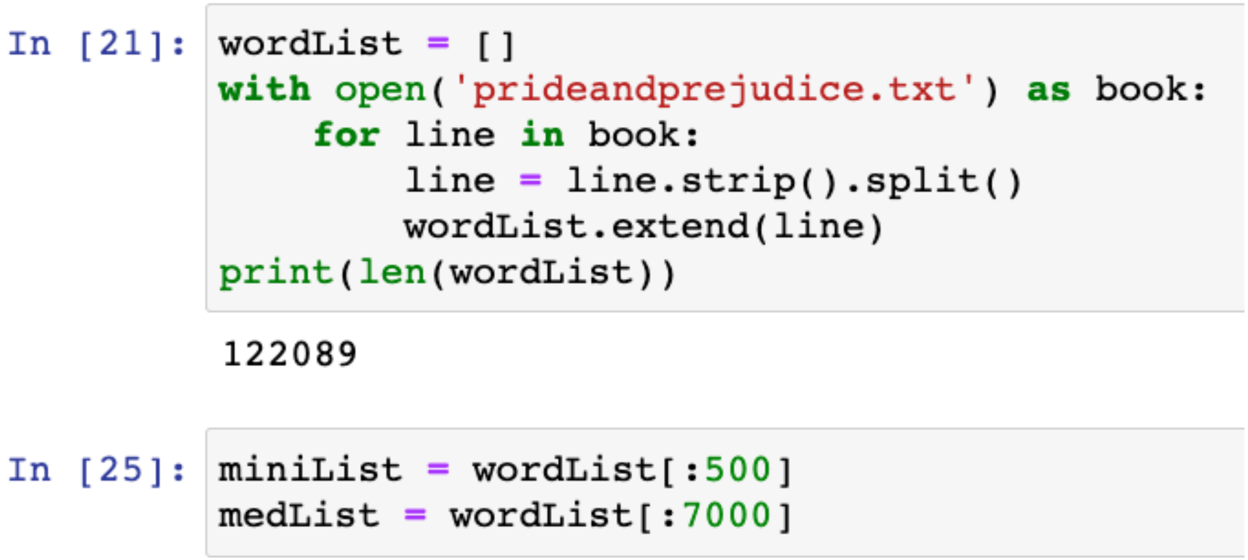
Merge Sort: Basic Idea

- If we split the list in half, sorting the left and right half are smaller versions of the same problem
- **Algorithm:**
    - **(Divide)** Recursively sort left and right half (O(log n))
    - **(Conquer)** Merge the sorted halves into a single sorted list (O(n))
    - (More info in extra slides at the end of this lecture!)



Selection vs Merge Sort in Practice

- Selection sort is $O(n^2)$ and merge sort is $O(n \log n)$ time
- But, how different is the performance of each in practice?
- Example: `wordList` is12,000 words in the book *Pride & Prejudice*
- `miniList` and `medList` are the first 500 and 7000 words respectively



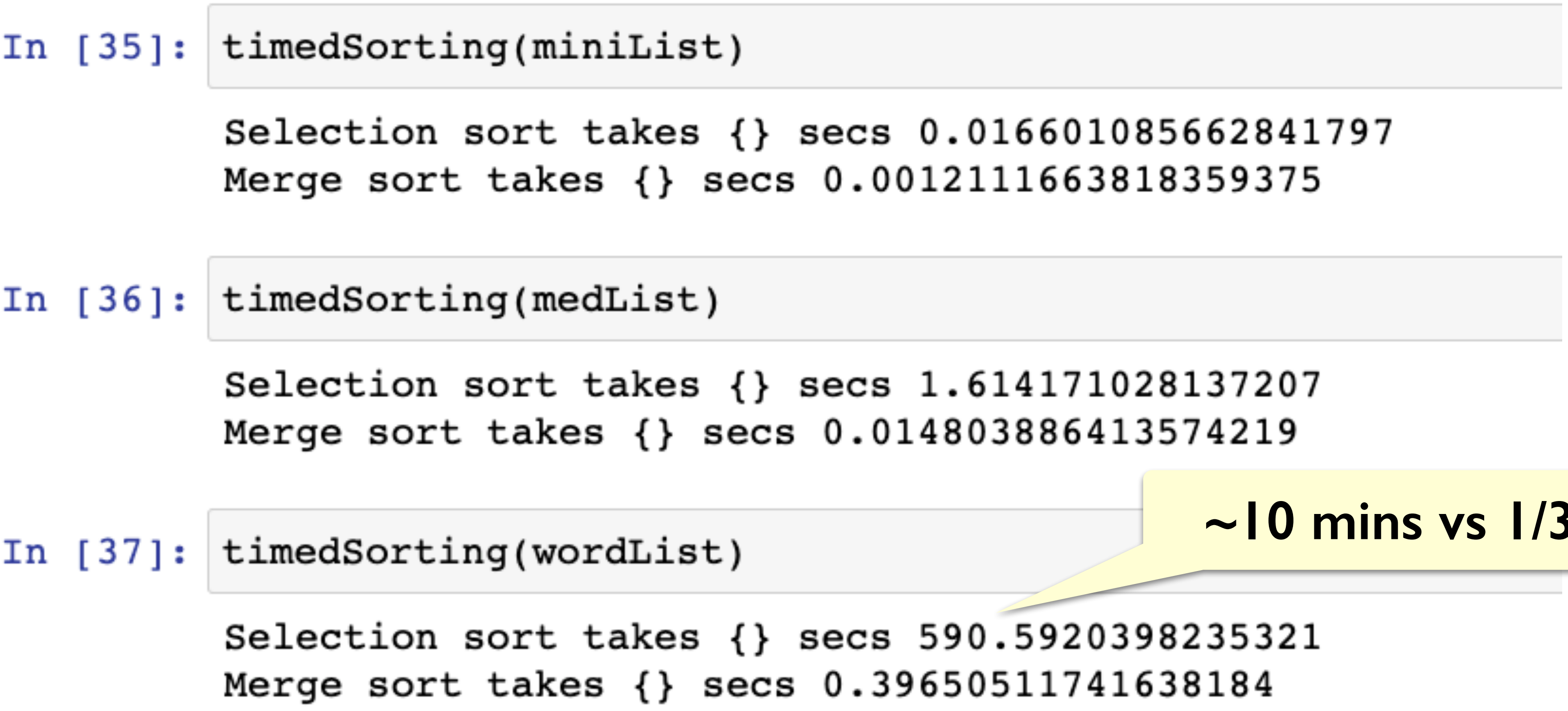
```
In [21]: wordList = []
         with open('prideandprejudice.txt') as book:
             for line in book:
                 line = line.strip().split()
                 wordList.extend(line)
         print(len(wordList))

         122089

In [25]: miniList = wordList[:500]
         medList = wordList[:7000]
```

Selection vs Merge Sort in Practice

- `miniList:  500 words`
- `medList:  7000 words`
- `wordList: ~12000 words`



```
In [35]: timedSorting(miniList)

         Selection sort takes {} secs 0.016601085662841797
         Merge sort takes {} secs 0.0012111663818359375

In [36]: timedSorting(medList)

         Selection sort takes {} secs 1.614171028137207
         Merge sort takes {} secs 0.014803886413574219

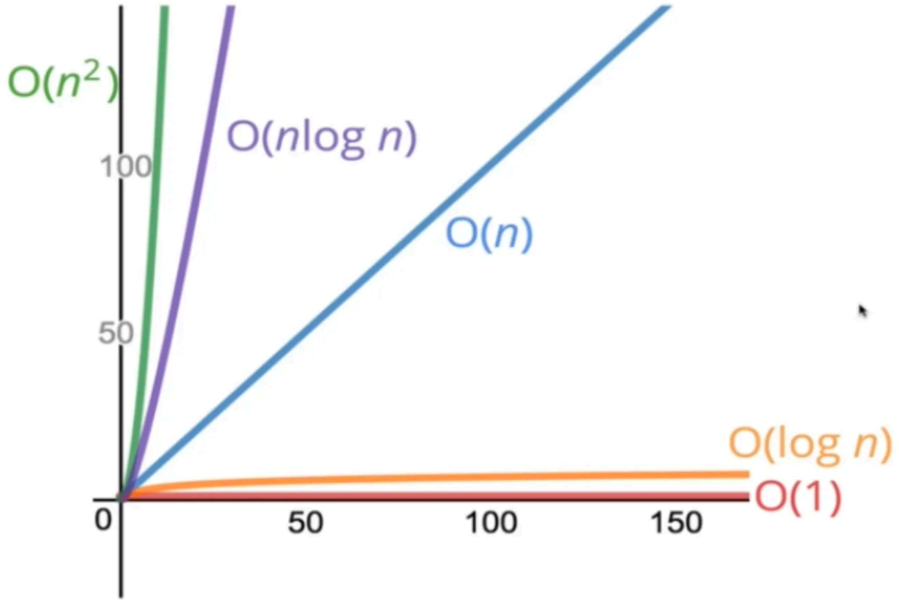
In [37]: timedSorting(wordList)

         Selection sort takes {} secs 590.5920398235321
         Merge sort takes {} secs 0.39650511741638184
```

~10 mins vs 1/3 sec!

Summary: Searching and Sorting

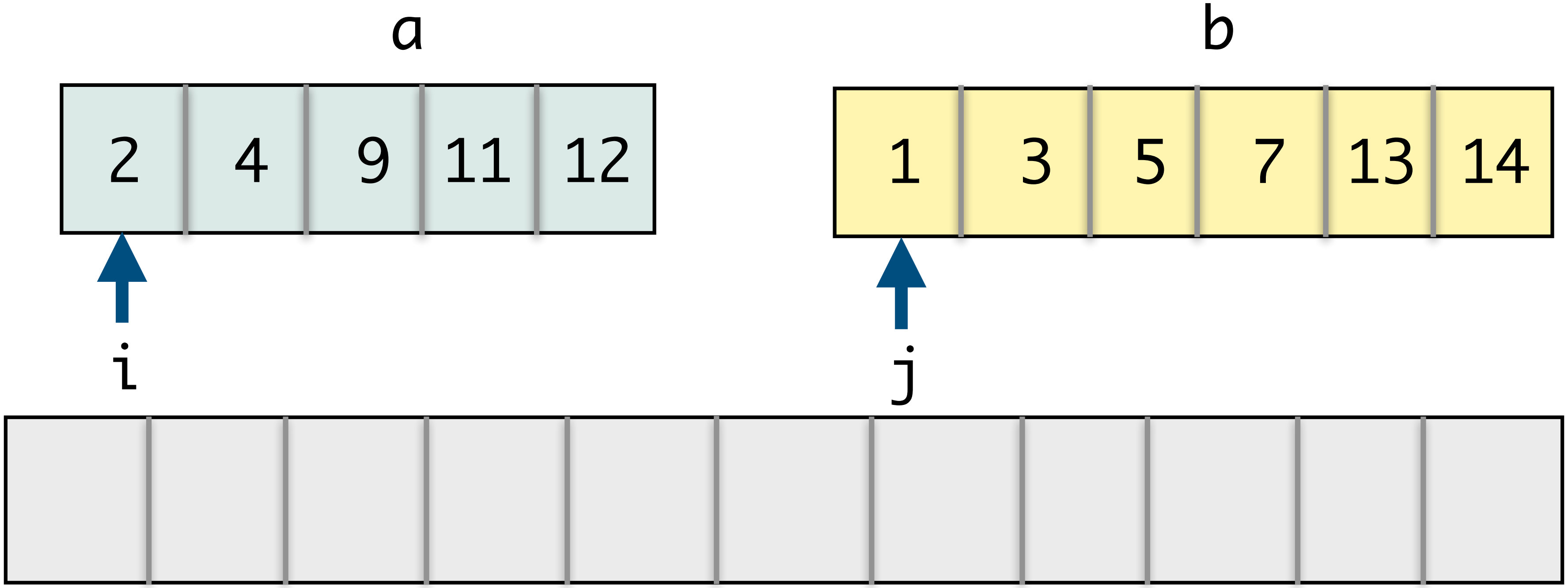
- We have seen algorithms that are
  - $O(\log n)$: binary search in a sorted list
  - $O(n)$: linear searching in an unsorted list
  - $O(n \log n)$: merge sort
  - $O(n^2)$: selection sort
- Important to think about efficiency when writing code!
- More about this in CS136!



Extra Slides

Merging Sorted Lists

- **Problem.** Given two sorted lists **a** and **b**, how quickly can we merge them into a single sorted list?



a

2	4	9	11	12

b

1	3	5	7	13	14

i

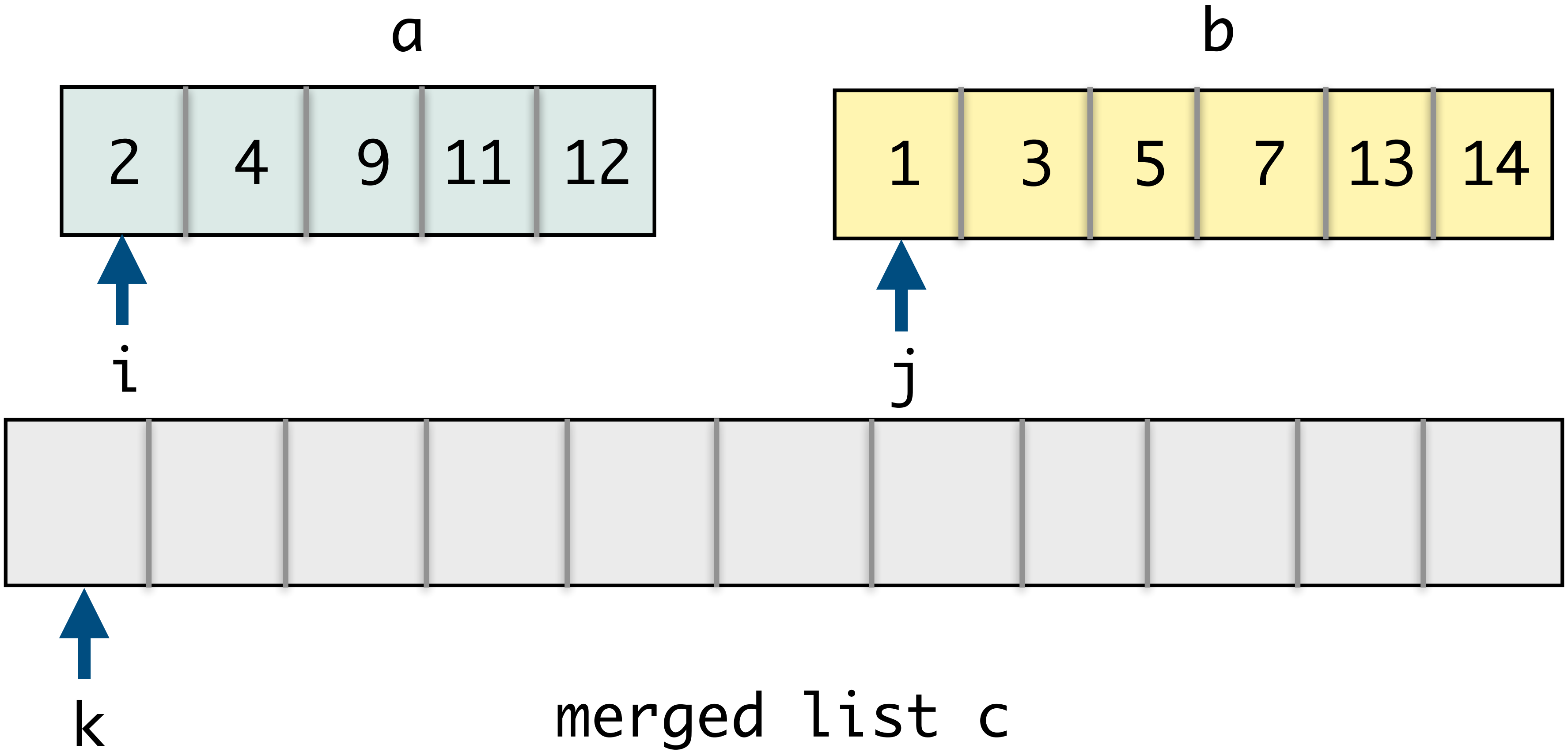
j

`merged list c`

Merging Sorted Lists

Is `a[i] <= b[j]` ?

- Yes, `a[i]` appended to `c`
- No, `b[j]` appended to `c`



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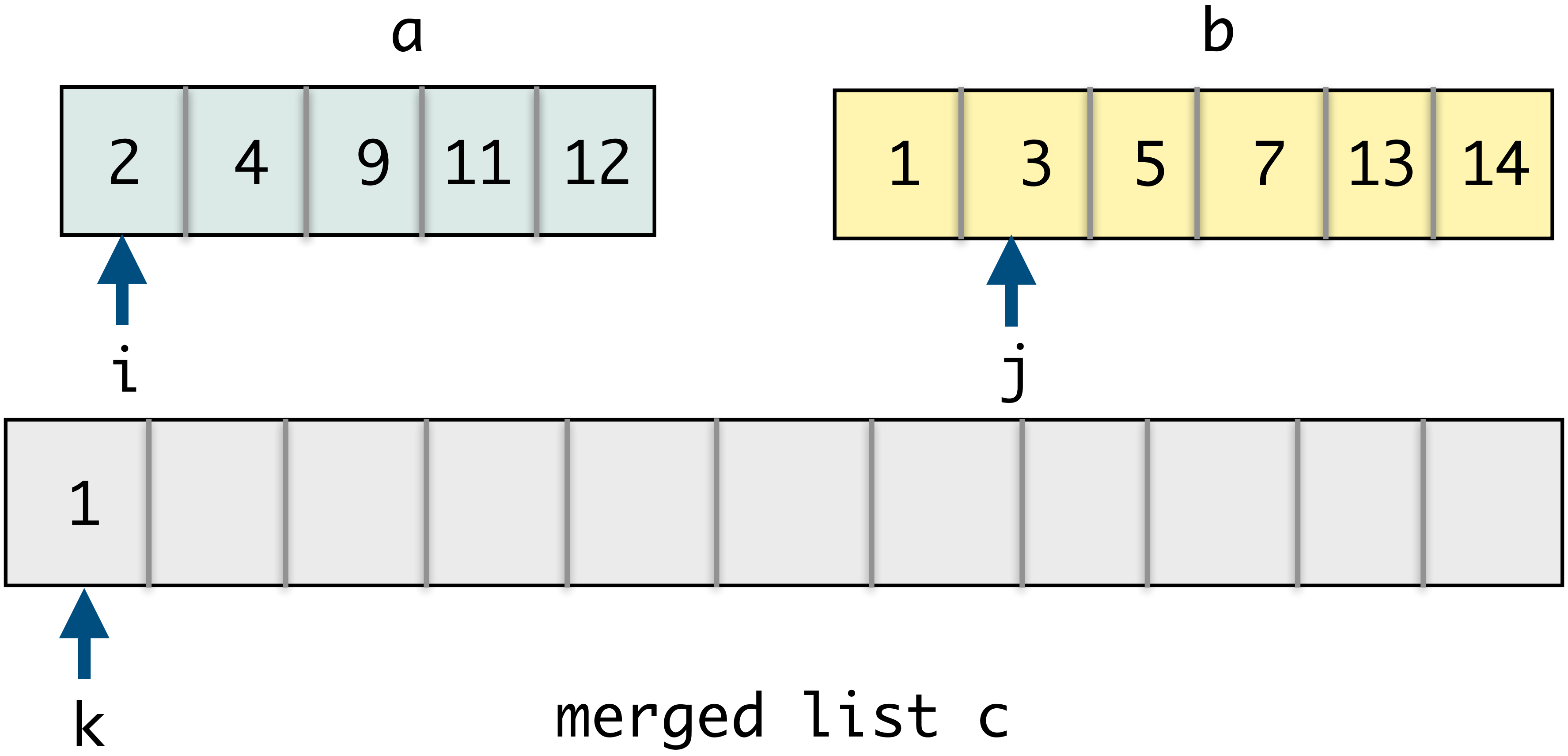

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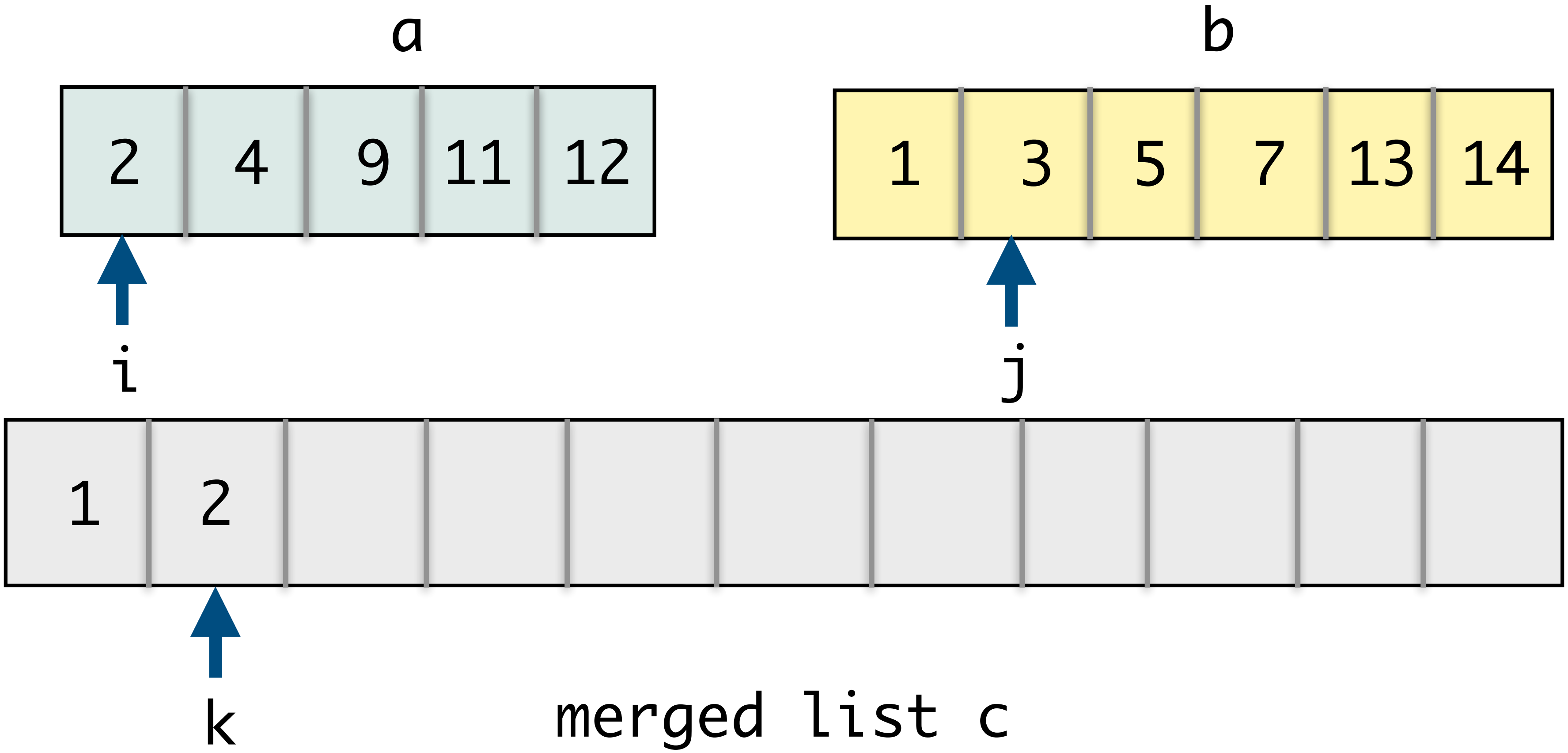

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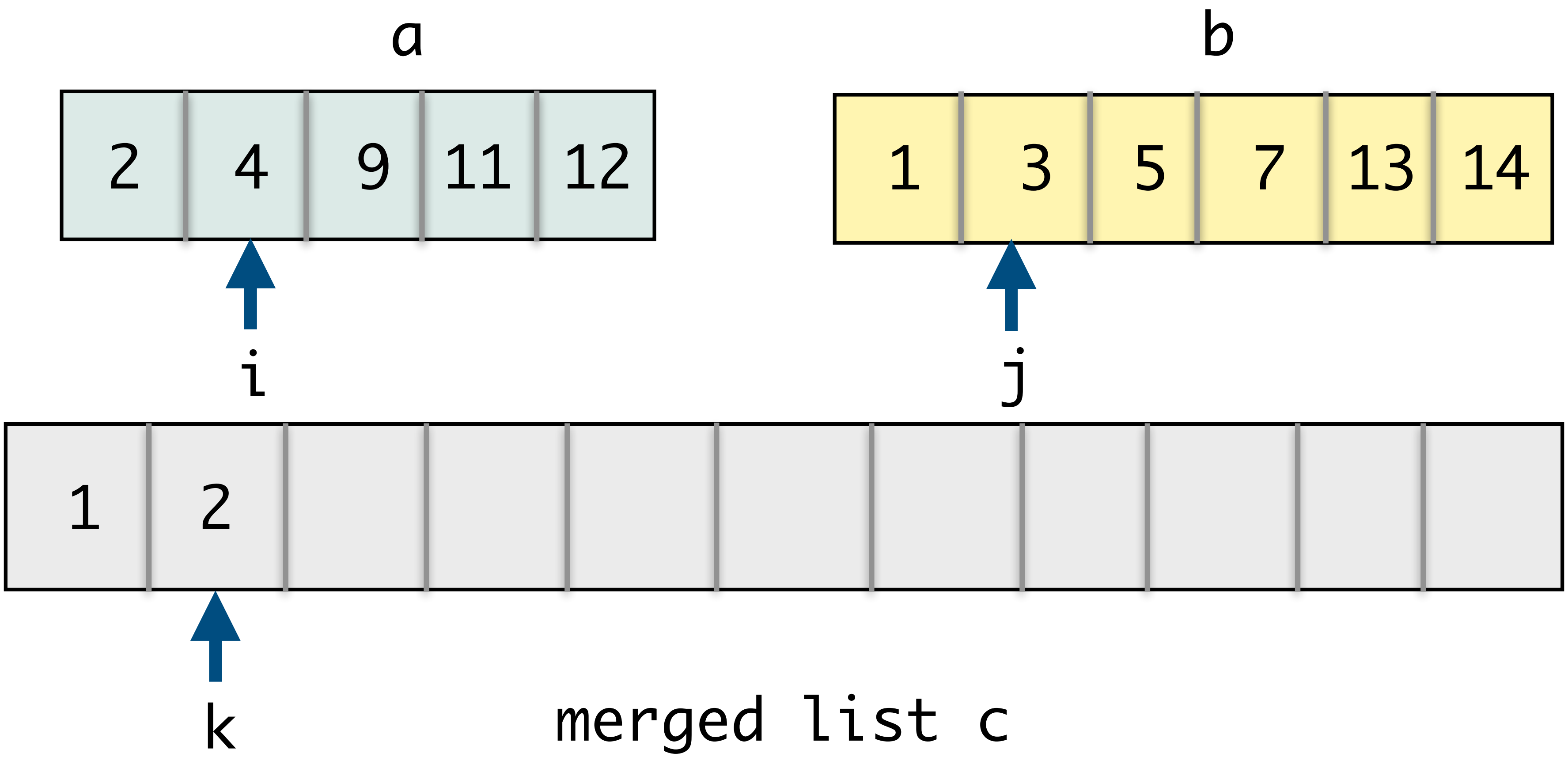

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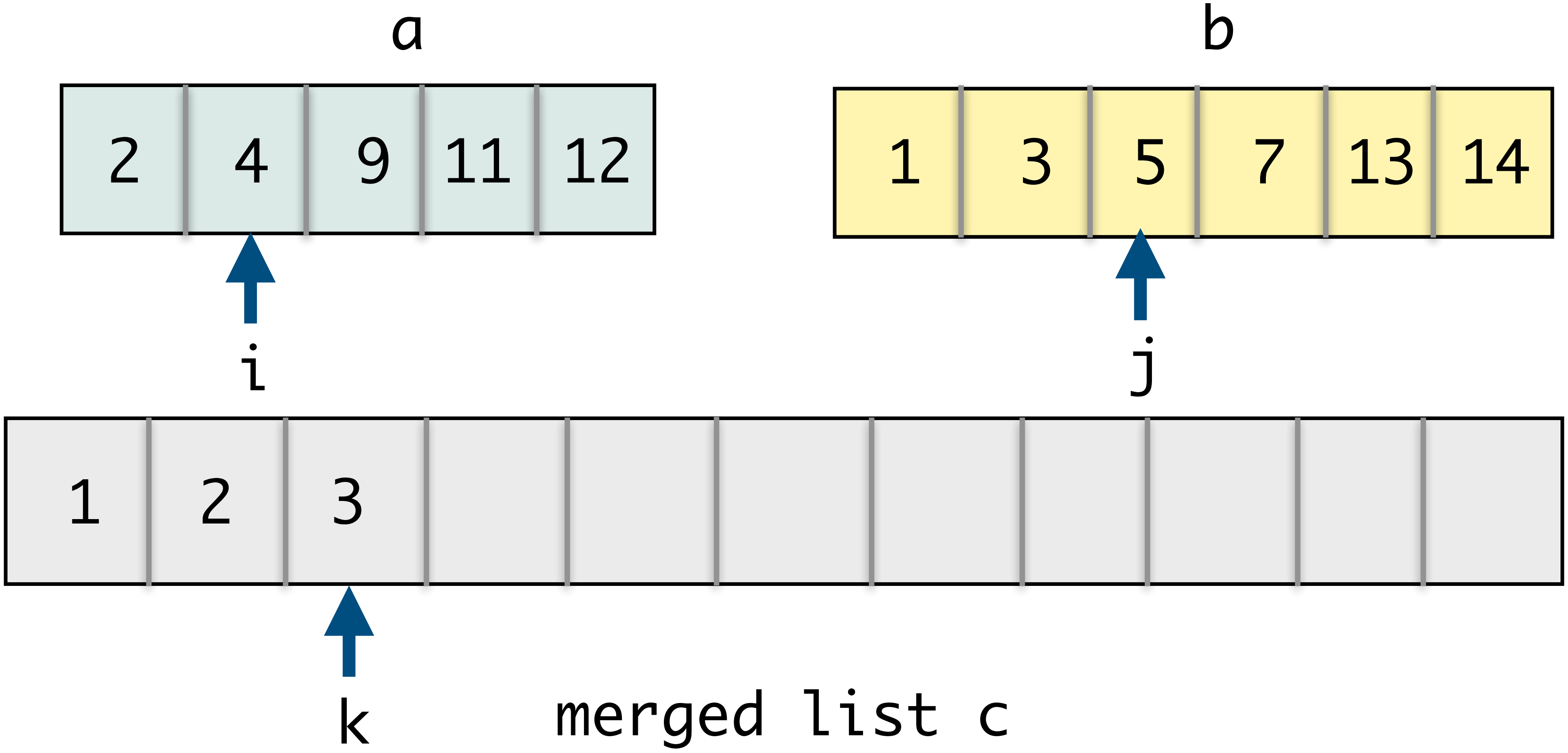

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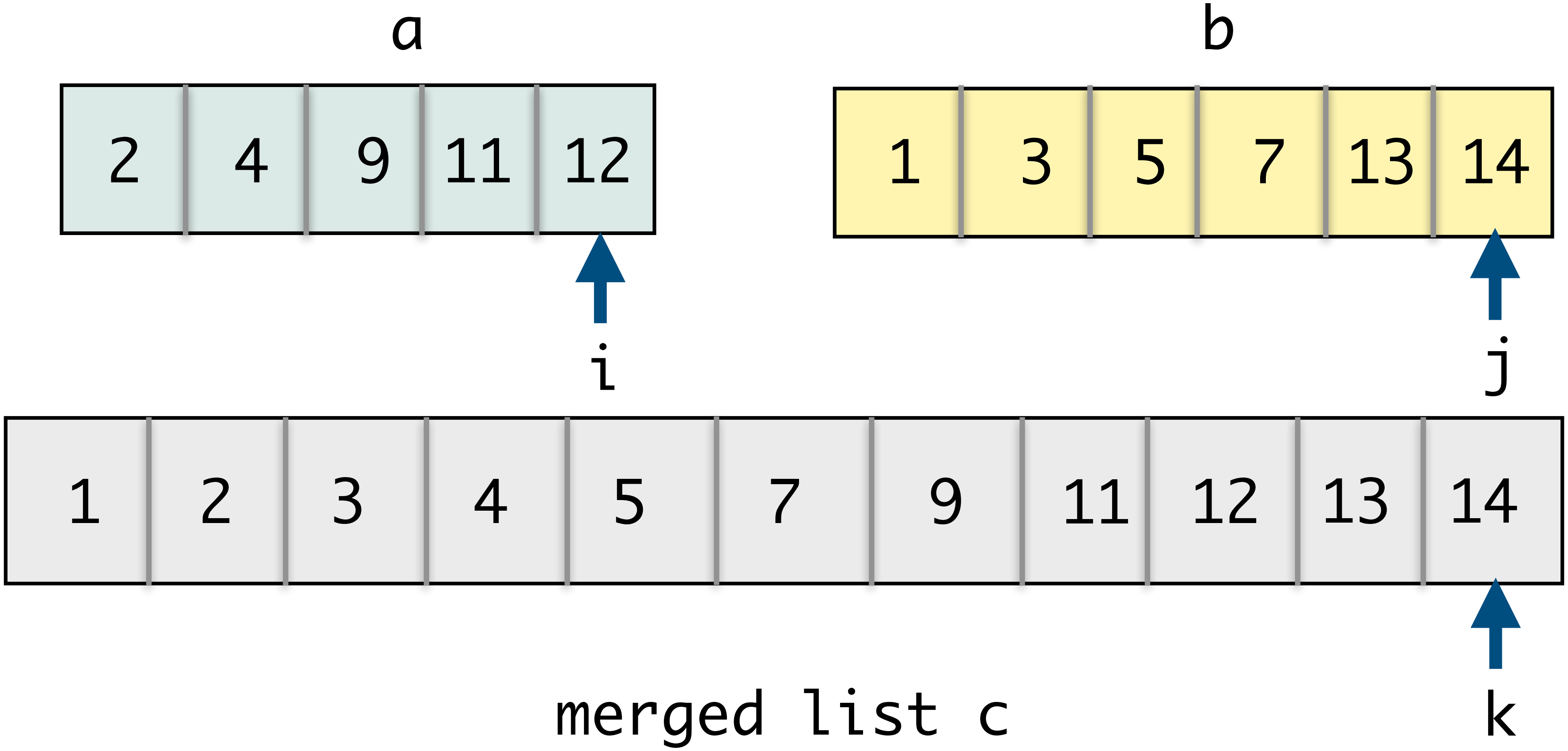
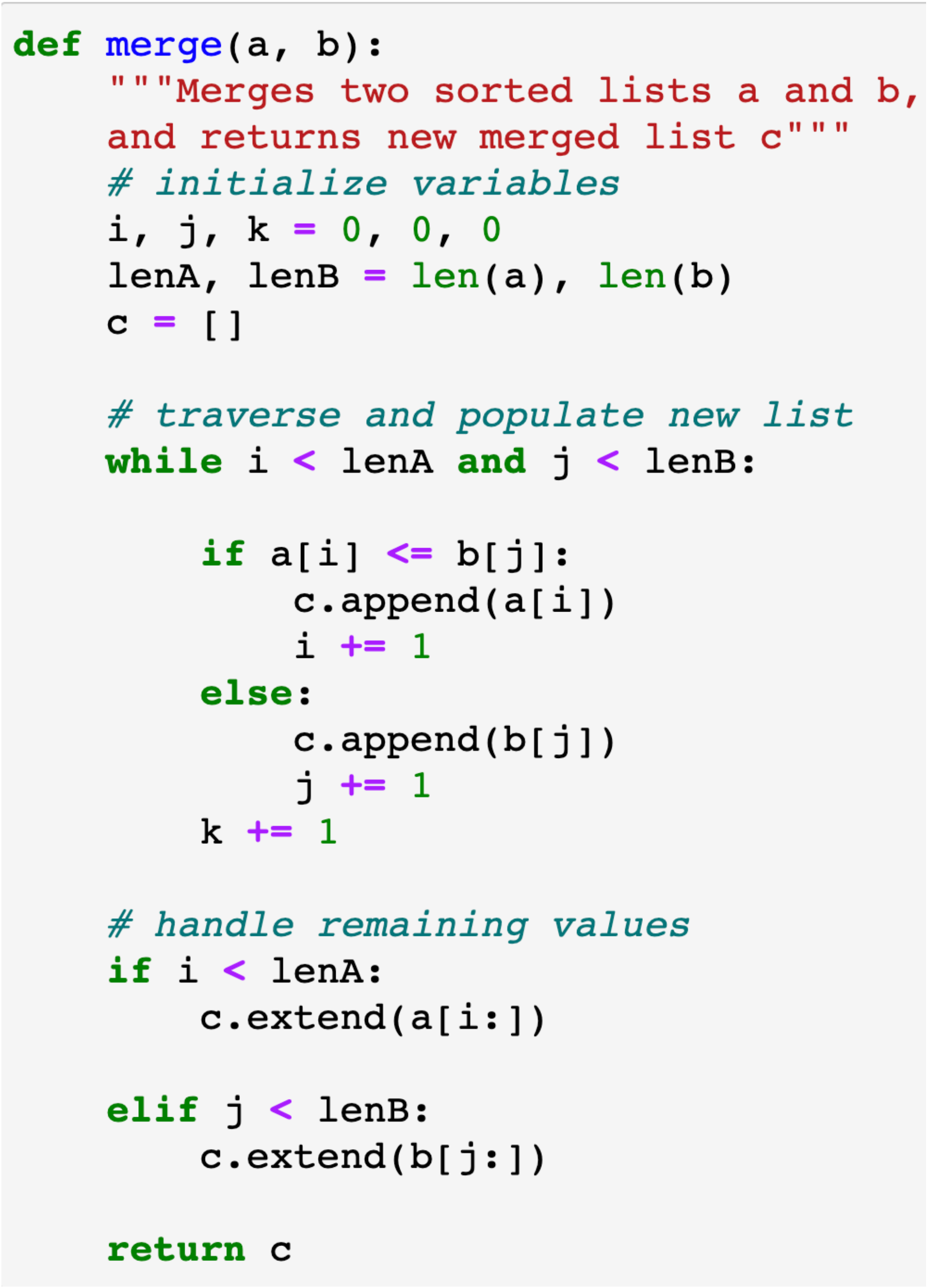


Merging Sorted Lists

- Walk through lists a, b, c maintaining current position of indices i, j, k
- Compare $a[i]$ and $b[j]$, whichever is smaller gets put in the spot of $c[k]$
- Merging two sorted lists into one is an $O(n)$ step algorithm!
- Can use this merge procedure to design our recursive merge sort algorithm!



```
def merge(a, b):
    """Merges two sorted lists a and b,
    and returns new merged list c"""
    # initialize variables
    i, j, k = 0, 0, 0
    lenA, lenB = len(a), len(b)
    c = []

    # traverse and populate new list
    while i < lenA and j < lenB:

        if a[i] <= b[j]:
            c.append(a[i])
            i += 1
        else:
            c.append(b[j])
            j += 1
        k += 1

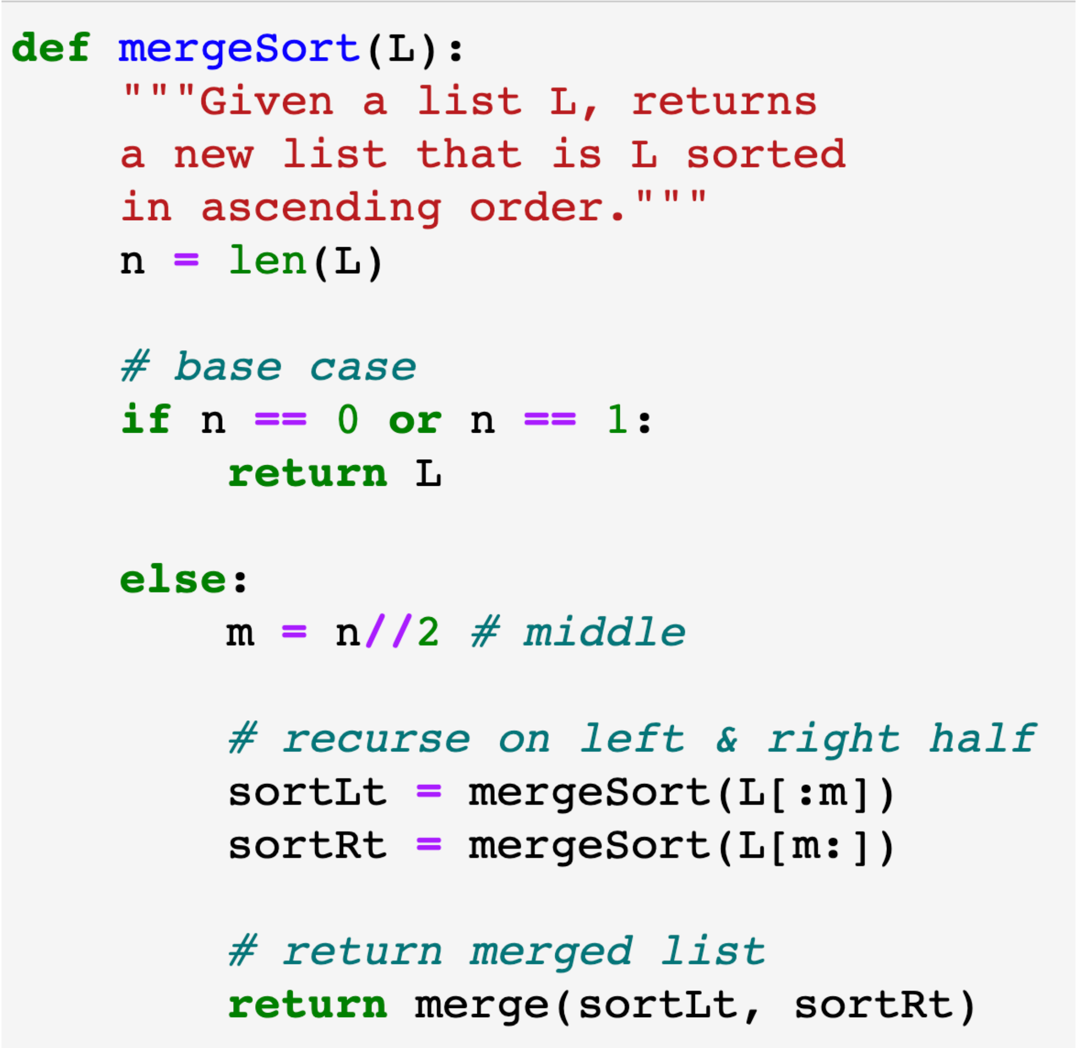
    # handle remaining values
    if i < lenA:
        c.extend(a[i:])

    elif j < lenB:
        c.extend(b[j:])

    return c
```

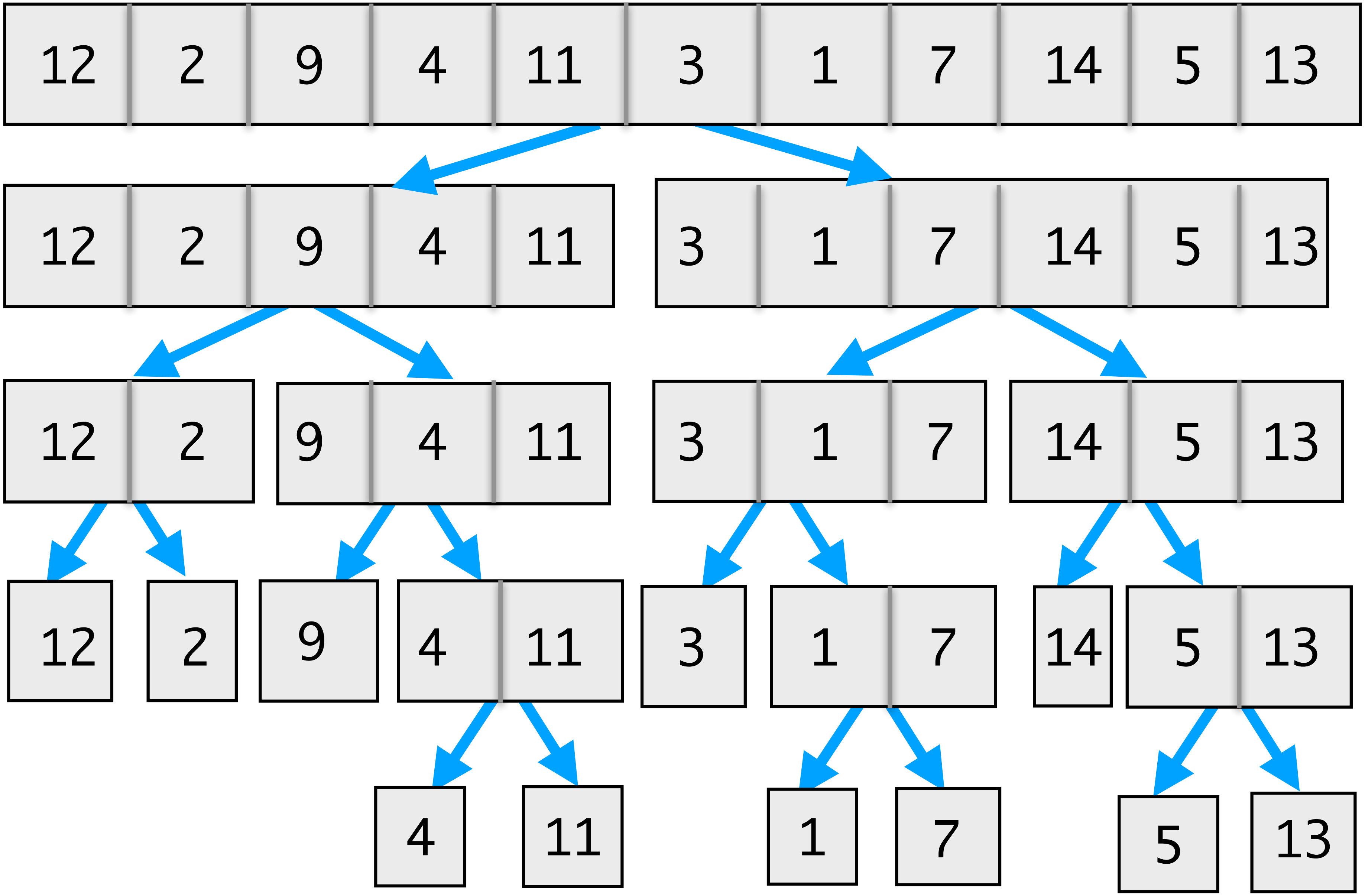
Merge Sort Algorithm

- **Base case:** If list is empty or contains a single element: it is already sorted
- **Recursive case:**
    - Recursively sort left and right halves
    - Merge the sorted lists into a single list and return it
- **Question:**
    - Where is the ***sorting*** actually taking place?



```python
def mergeSort(L):
    """Given a list L, returns
    a new list that is L sorted
    in ascending order."""
    n = len(L)

    # base case
    if n == 0 or n == 1:
        return L

    else:
        m = n//2 # middle

        # recurse on left & right half
        sortLt = mergeSort(L[:m])
        sortRt = mergeSort(L[m:])

        # return merged list
        return merge(sortLt, sortRt)
```

Merge Sort Example



Merge Sort Example