CS 134:
Searching

Announcements & Logistics

- **Lab 7** feedback returned!
- **Lab 8** feedback coming soon!
- **Lab 9 part 1 feedback returned**: let us know if you have any questions!
- **Lab 9 Boggle**
  - **Completed version of all classes** due next Wed/Thur
  - Make sure you thoroughly test your code
- Colloquium today: 2:35 in Wege

Do You Have Any Questions?

Last Time: Iterators

- Learned about **iterables** and **iterators**
- An object is considered **iterable** if it supports the `iter()` function (special method `__iter__` is defined): e.g, lists, strings, tuples
  - When an **iterable** is passed to the `iter()` function, it creates and returns an **iterator**
  - An **iterator** object can generate values **on demand**
    - **Supports the** `next()` function (and `__next__` method) which simply provides the "next" value in the sequence

Today and Next Week

- Briefly introduce how we measure efficiency in Computer Science
- Analyze the efficiency of some of our algorithms and data structures
- Next Monday:
    - Evaluate sorting algorithms and their efficiency
- Last 5 classes: Introduction to Java (and Python review)
    - Computational thinking and logic stays the same across programming languages
    - We will focus on how the two languages are different in their syntax and structure

Measuring Efficiency

- How do we measure the efficiency of our program?
  - We want programs that run "fast"
  - But what do we mean by that?
- One idea: use a stopwatch to see how long it takes
  - Is this a good method?
  - What is the stopwatch really measuring?
  - How long does this piece of code takes **on this machine on this particular input.**
- Machine (and input) dependent
  - We want to evaluate our ***program's efficiency***, not the machine's speed
- Cannot make any general conclusions using this approach
  - Might not tell us how fast the program runs on different inputs/machines

Efficiency Metric: Goals

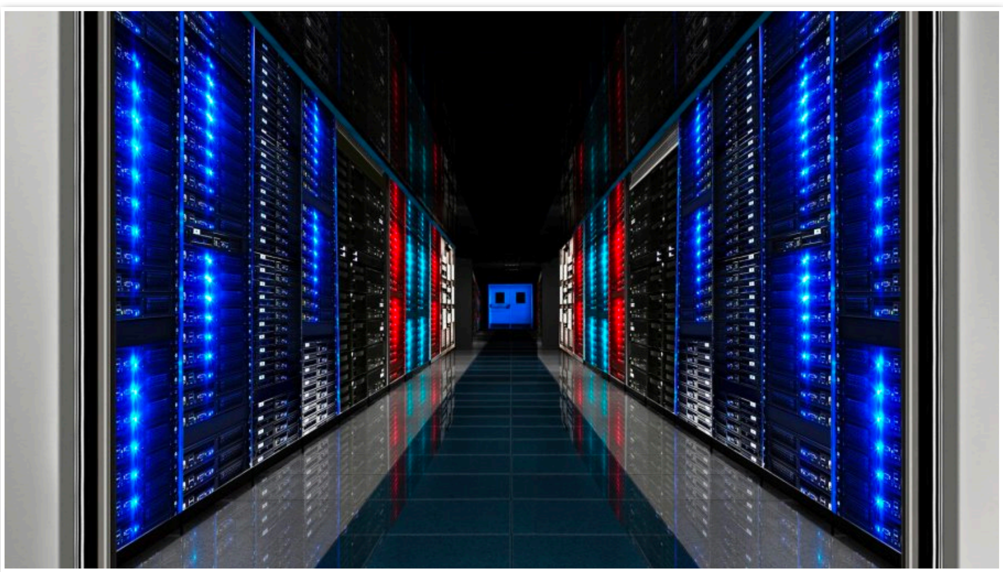
We want a method to evaluate efficiency that:

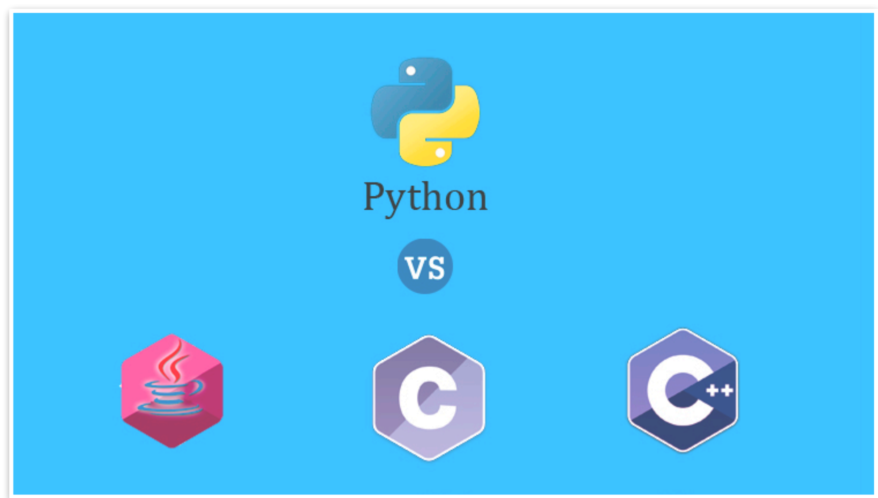
- **Is machine and language independent**
  - Analyze the ***algorithm*** (problem-solving approach)
- **Provides guarantees that hold for different types of inputs**
  - Some inputs may be "easy" to work with while others are not
- **Captures the dependence on input *size***
  - Determine how the performance "scales" when the input gets bigger
- **Captures the right level of specificity**
  - We don't want to be too specific (cumbersome)
  - Measure things that matter, ignore what doesn't

Platform/Language Independent

Machine and language independence

- We want to evaluate how good the algorithm is, rather than how good the machine or implementation is
- Basic idea: Count the number of steps taken by the algorithm
- Sometimes referred to as the "running time"



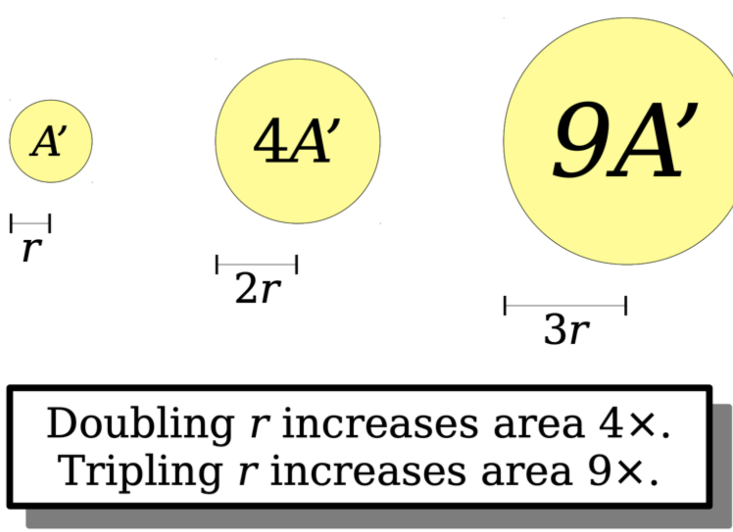


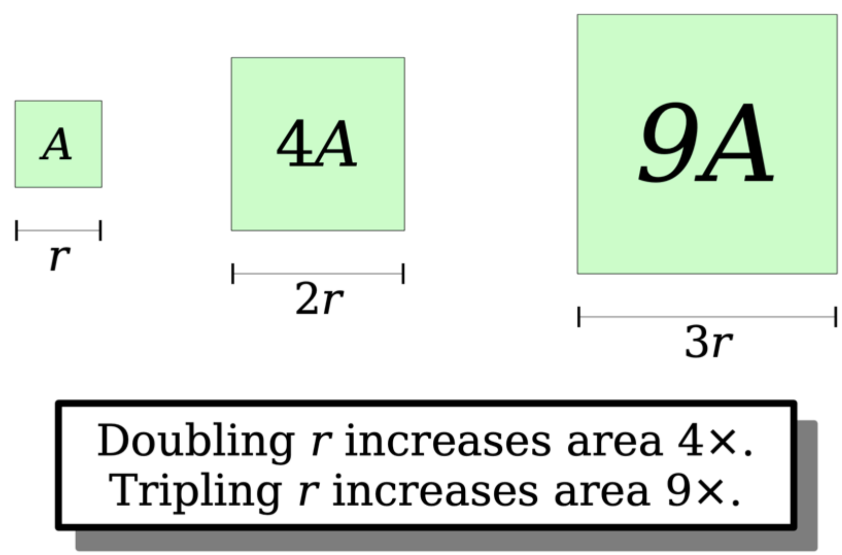
Worst-Case Analysis

- We can't just analyze our algorithm on a few inputs and declare victory
  - **Best case.** Minimum number of steps taken over all possible inputs of a given size
  - **Average case.** Average number of steps taken over all possible inputs of a given size
  - **Worst case.** Maximum number of steps taken over all possible inputs of a given size.
- Benefit of wort case analysis:
  - Regardless of input size, we can conclude that the algorithm always does *at least as well as* the pessimistic analysis

Dependence on Input Size

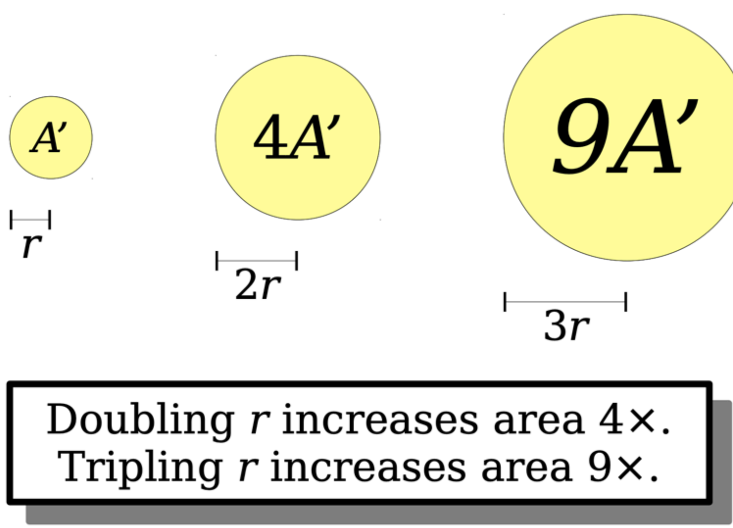
- We generally don't care about performance on "small inputs"
- Instead we care about "the rate at which the completion time grows" with respect to the input size
- For example, consider the area of a square or circle: while the formula for each is different, they both grow at the same rate wrt radius
    - doubling radius increases area by 4x, tripling increases by 9x

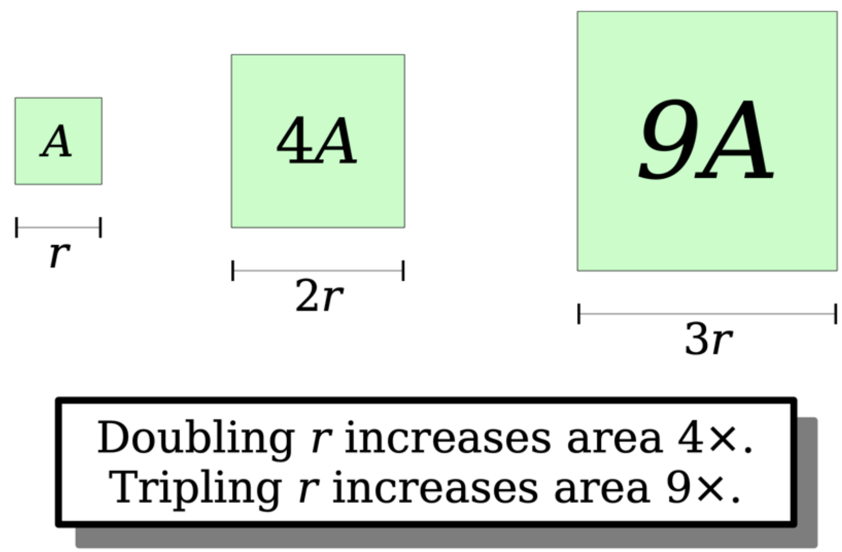




Dependence on Input Size: Big-O

- Big-O notation in Computer Science is a way of quantifying (in fact, upper bounding) the growth rate of algorithms/functions wrt input size
- For example:
  - A square of side length r has area $O(r^2)$.
  - A circle of radius r has area $O(r^2)$.





Dependence on Input Size: Big-O

- Big-O notation captures the rate at which which the number of steps taken by the algorithm grows wrt size of input n, "as n gets large"
- Not precise by design, it ignores information about:
  - Constants (that do not depend on input size n), e.g. $100n = O(n)$
  - Lower-order terms: terms that contribute to the growth but are not dominant: $O(n^2 + n + 10) = O(n^2)$
- Powerful tool for predicting performance behavior: focuses on what matters, ignores the rest
- Separates fundamental improvements from smaller optimizations
- We won't study this notion formally: covered in CS136 and CS256!

Understanding Big-O

- Notation: n often denotes the number of elements (size)
- **Constant time** or $O(1)$: when an operation does not depend on the number of elements, e.g.
  - Addition/subtraction/multiplication of two values, or defining a variable etc are all constant time
- **Linear time** or $O(n)$: when an operation requires time proportional to the number of elements, e.g.:

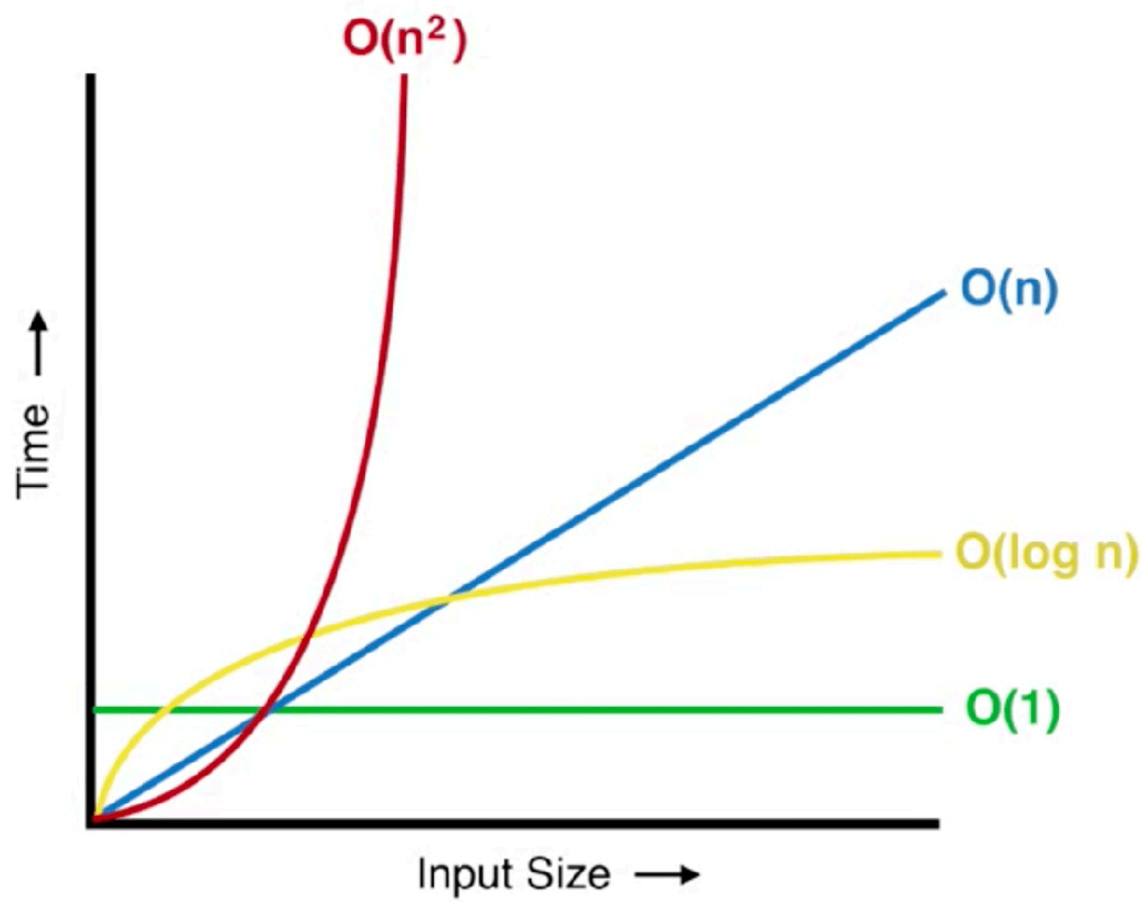
```
for item in seq:
   <do something>
```

- **Quadratic time** or $O(n^2)$: nested loops are often quadratic, e.g.,

```
for i in range(n):
   for j in range(n):
        <do something>
```

Big-O: Common Functions

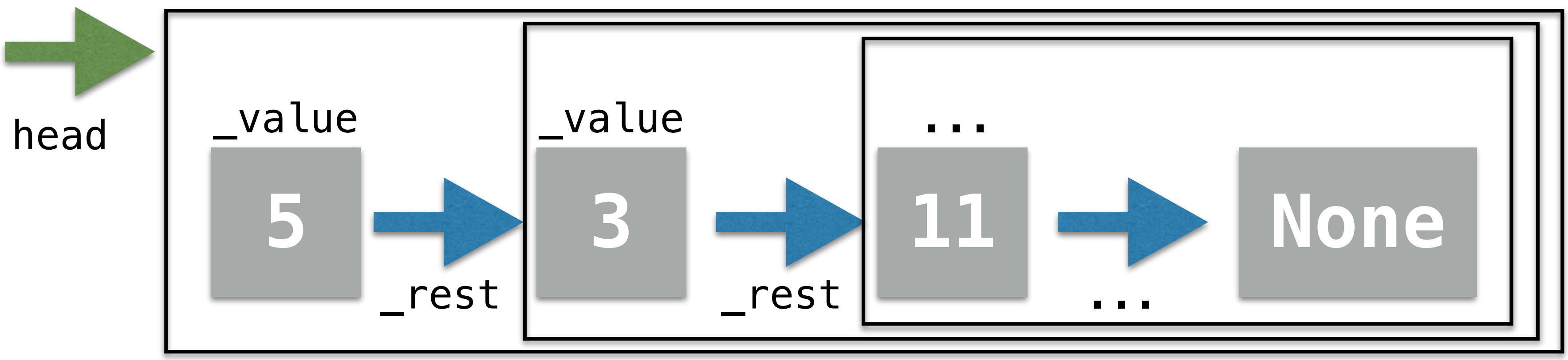
- Notation: n often denotes the number of elements (size)
- Our goal: understand efficiency of some algorithms at a high level



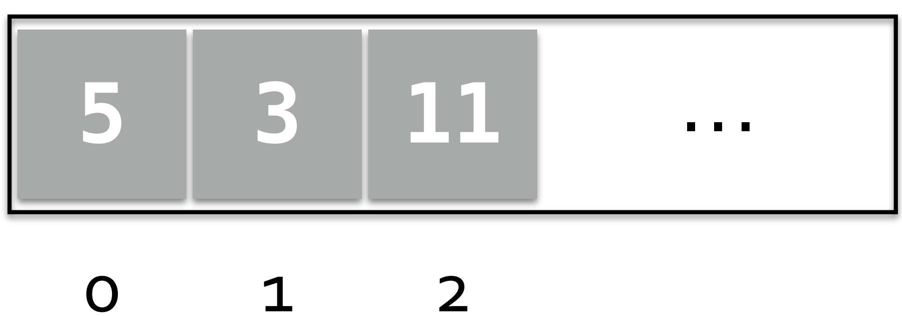
Lists vs Linked Lists:
Efficiency Trade Offs

Lists vs Linked Lists

- **Linked Lists**: "pointer-based" data structure, items need not be contiguous in memory

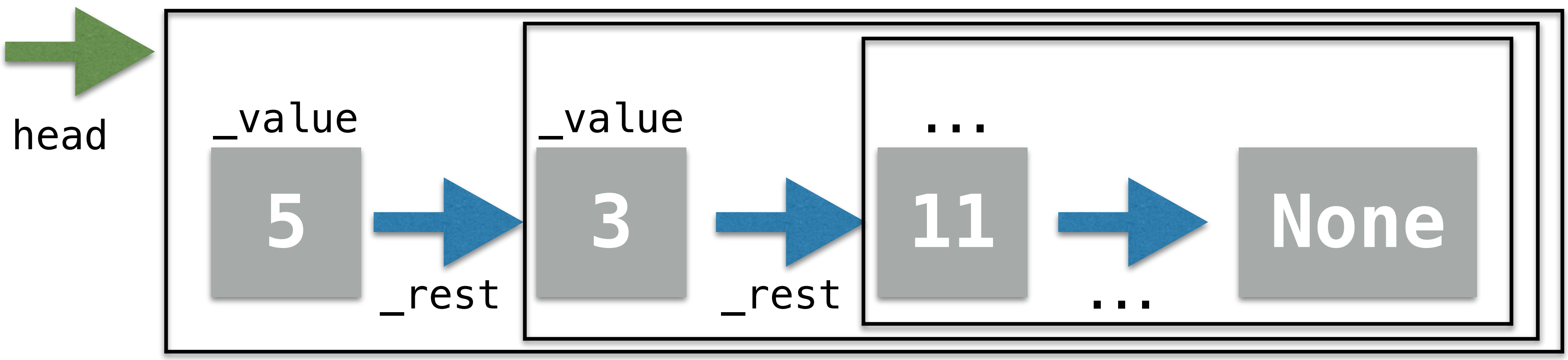


- **Lists:** index-based data structure (sometimes called **arrays**), items are always stored contiguously in memory

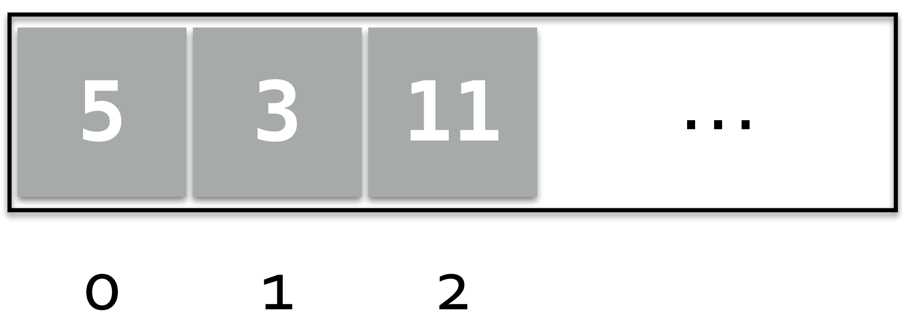


Lists vs Linked Lists

- **Linked Lists**: Can grow and shrink on the fly: do not need to know size at the time of creation (therefore no wasted space!)

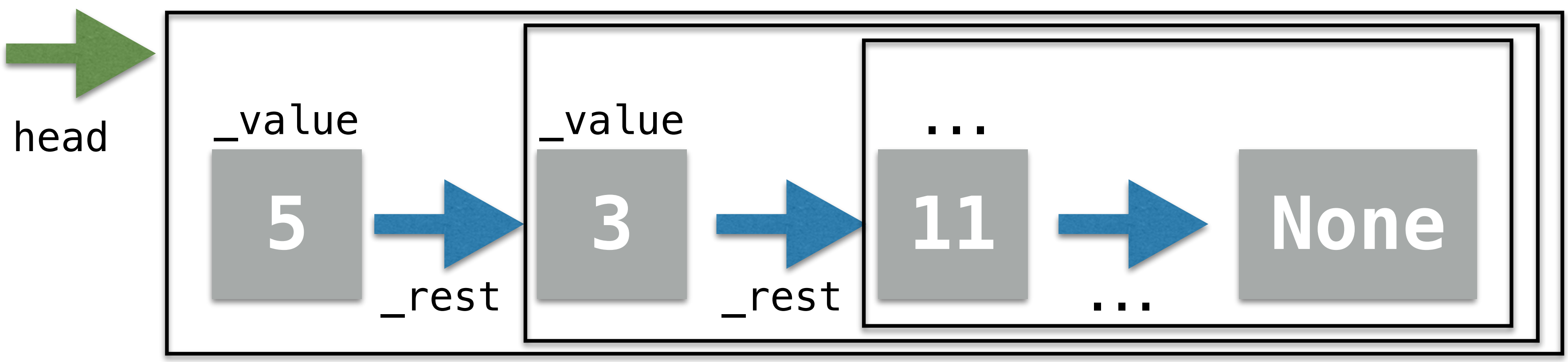


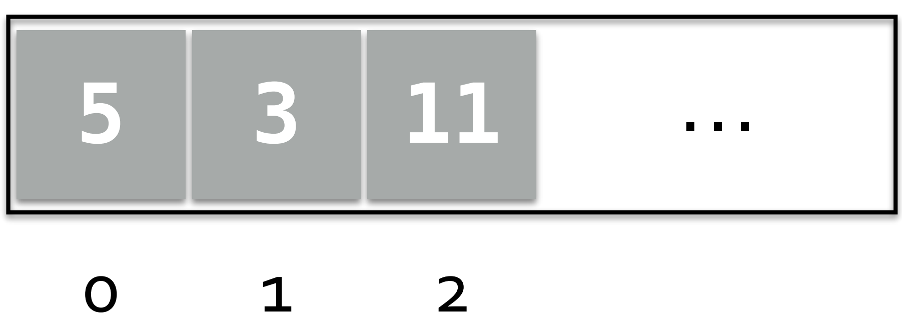
- **Lists:** Need to know size (or use some default value) at the time of creation, can waste space by leaving room for future insertions



An Aside: What exactly is Python's list?

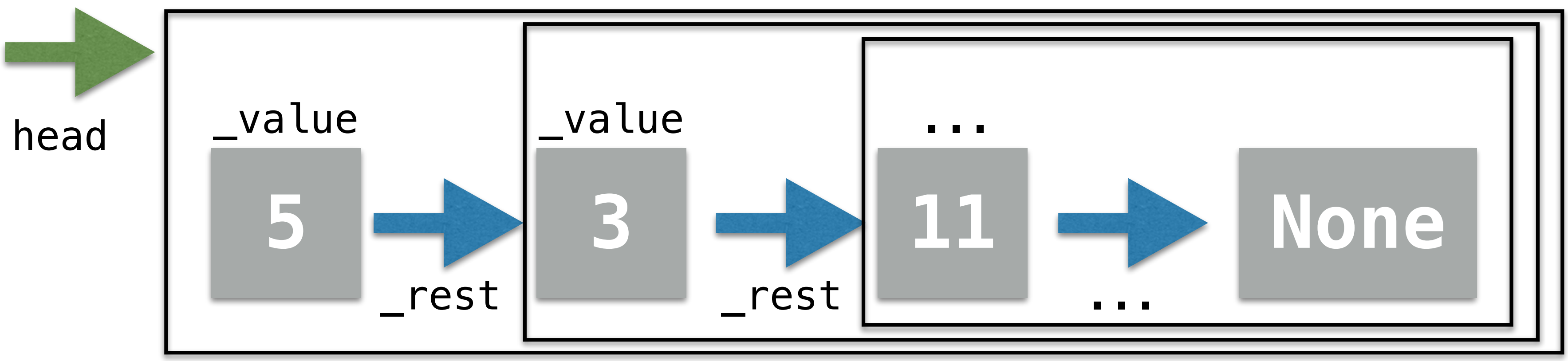
- It's complicated: Python's list implementation is a hybrid
- For today's lecture, we will assume its an array-based structure (lower picture)

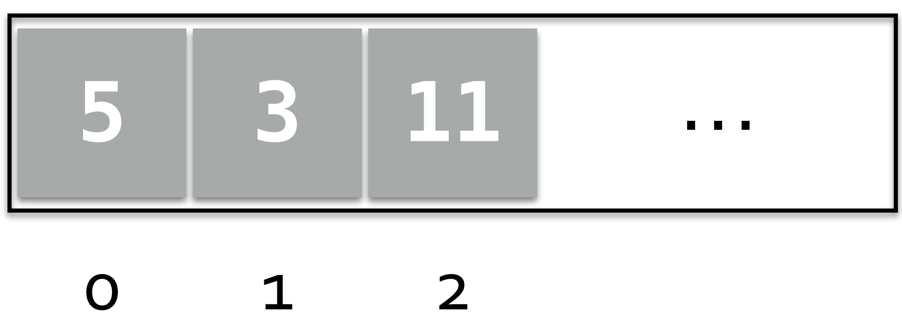




Array vs Linked Lists

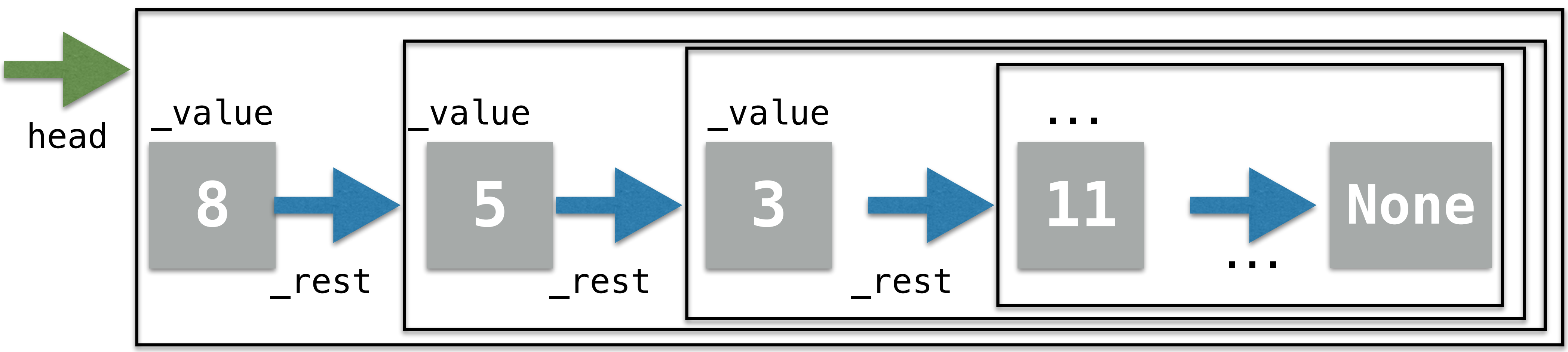
- Inserts at the beginning: which one is better?





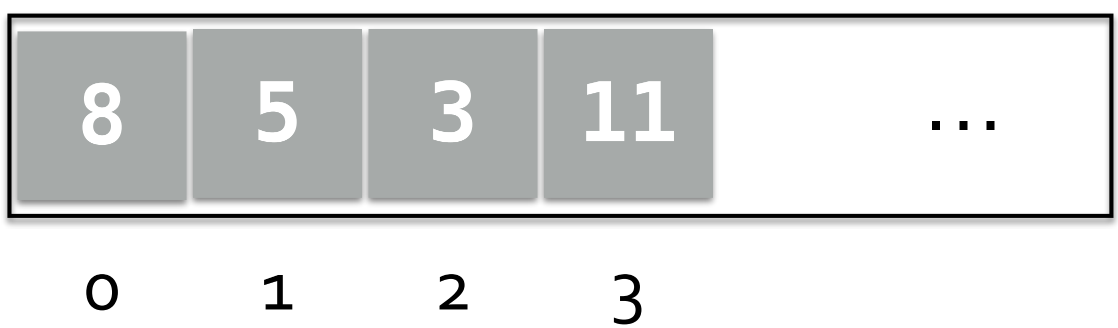
Array vs Linked Lists

- Linked list steps:
    - Point head to new element
    - Point rest of new element to old list
    - These steps don't depend on size of list
    - Therefore, run-time is **constant**, that is, $O(1)$ time



Array vs Linked Lists

- Now consider an array-based list
- To insert at index 0, we need to shift every element over by one spot
  - This takes time proportional to the size: linear time or $O(n)$
- So when are arrays more efficient?
  - When **indexing** elements: they give **direct access** $O(1)$
  - Linked list: we need to traverse the list to get to the element $O(n)$



So Which is Better?

- It depends!
- **Time-space tradeoff**: try to find a balance between ***time efficiency*** and ***space efficiency***
- Think about what list operations are required the most for your program
- Choose accordingly

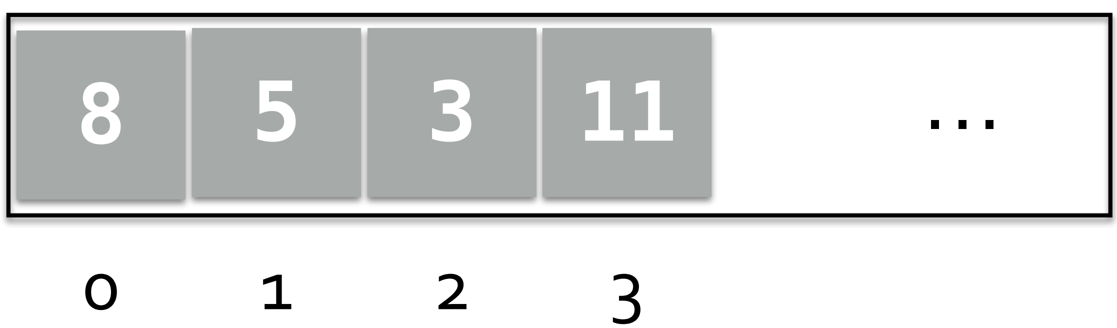
Searching in an Array

Searching in an Array

- Let us discuss how quickly we can search for an item in an array-based list

```
def linearSearch(val, myList):
    for elem in myList:
        if elem == val:
            return True
    return False
```

Might return early if val is first item in myList, but we are interested in the **worst case analysis**; this happens if val is not in the myList at all



Searching in an Array

- In the worst case, we have to walk through the entire sequence
- Takes linear time, or $O(n)$

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def linearSearch(val, myList):
    for elem in myList:
        if elem == val:
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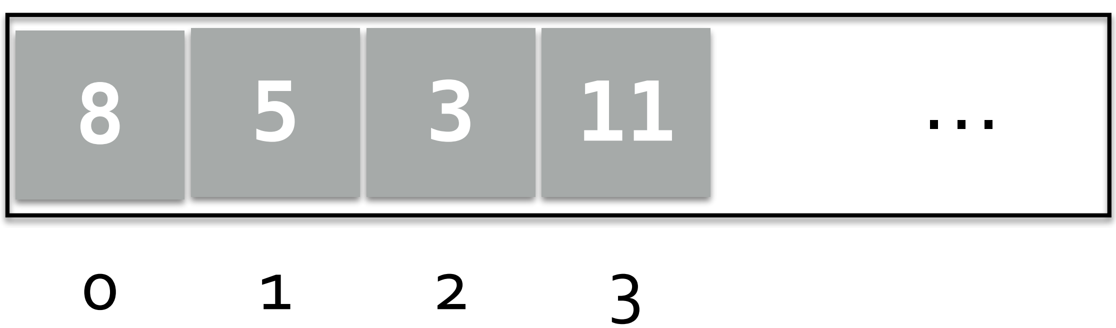
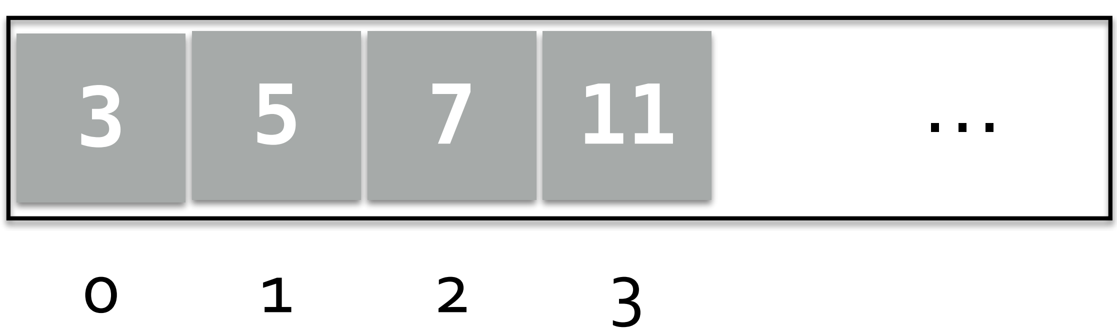


Searching in an Array

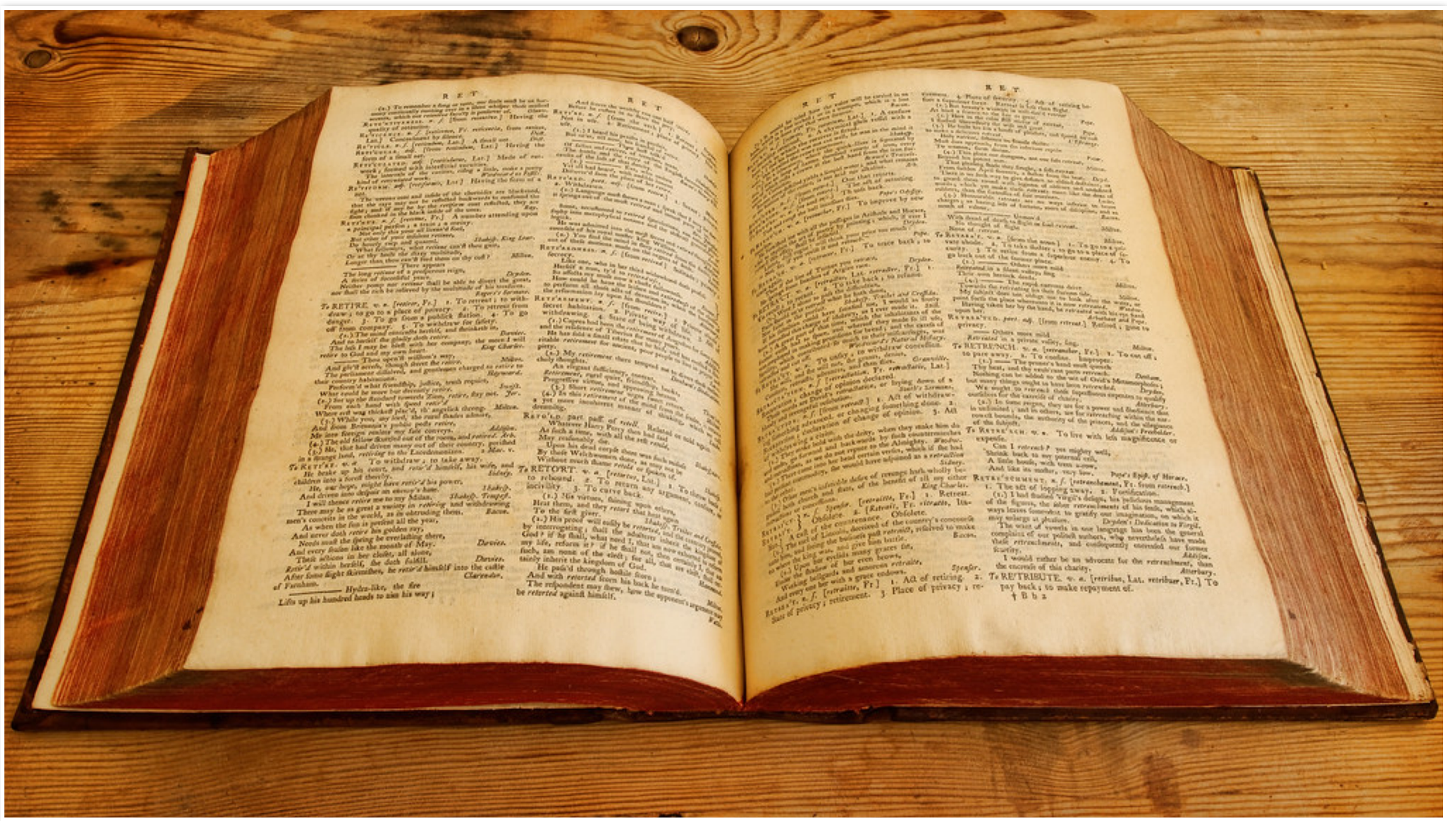
- Can we do better?
    - Not if the elements are in arbitrary order
- What if the sequence is **sorted**?
    - Can we utilize this somehow and search more efficiently?

How do we search for an item (say 10) in a **sorted** array?



Example: Dictionary

- How do we look up a word in a (physical) dictionary?
- Words are listed in alphabetical order

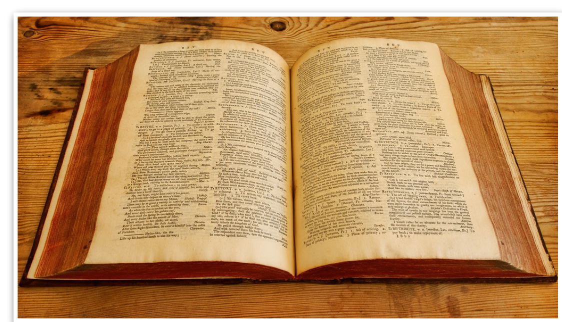


Searching for Word in Dictionary

- Look at the (approximately) middle page for our word
- If we find our word, great!
- Otherwise:
  - If our word is **later** in alphabetical order than the words on the page, look for the word **between the middle page and the last page**
  - If our word is **earlier** in alphabetical order, look for the word **between the middle page and the first page**

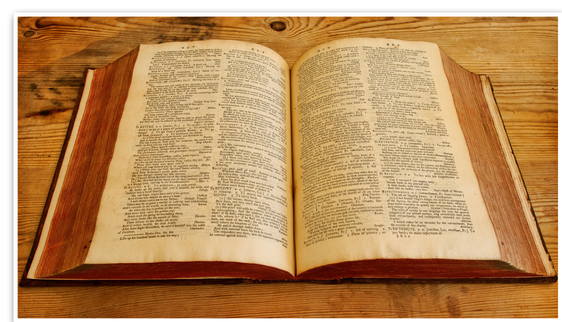
How Good is This Method?

- **Goal:** Analyze # pages we need to look at until we find the word
- We want the worst case: possible to get lucky and find the word right on the middle page, but we don't want to consider luck!
- Each time we look at the "middle" of the remaining pages, the number of pages we need to look at is divided by 2
- A 1024-page dictionary requires at most 11 lookups:
  1024 pages, < 512, <256, <128, <64, <32, <16, <8, <4, <2, <1 page.
- Only needed to look at 11 pages out of 1024 !
- Challenge: What if we have an n page dictionary, what function of n characterizes the (worst-case) number of lookups?



Logarithms: our favorite function

- Logarithms are the inverse function to exponentiation
- $\log_2 n$ describes the exponent to which 2 must be raised to produce n
- That is, $2^{\log_2 n} = n$
- Alternatively:
  - $\log_2 n$ (essentially) describes the number of times n must be divided by 2 to reduce it to below 1
- For us, here's the important takeaway:
  - How many times can we divide n by 2 until we get down to 1
  - $\approx \log_2 n$



Binary Search

- The **recursive search algorithm** we described to search in a sorted array is called **binary search**
- It is much, much more efficient than a **linear search:** $O(\log n)$ time
  - **Note:** $\log n$ grows much more slowly compared to n as n gets large
- Lets implement this technique

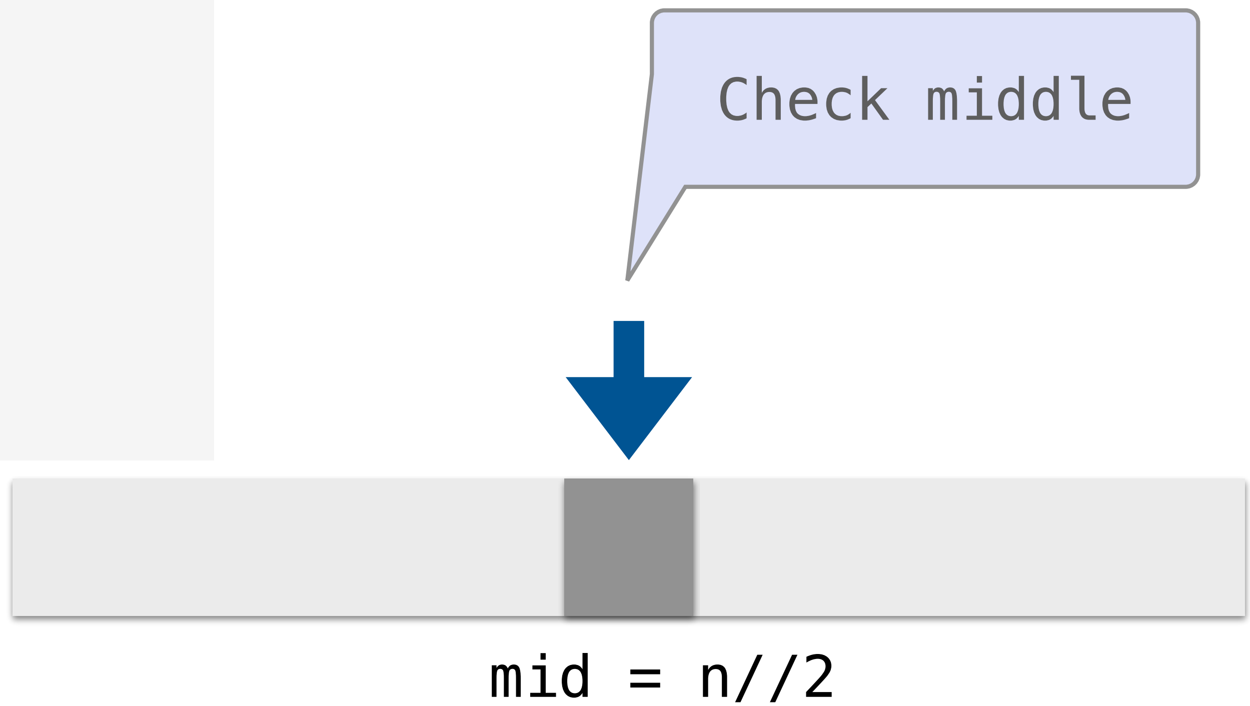
```
def binarySearch(aList, item):
    """Assume aList is sorted.
    If item is in aList, return True;
    else return False."""
    pass
```

Binary Search

- Base cases? When are we done?
  - If list is too small (or empty)
  - If item is the middle element

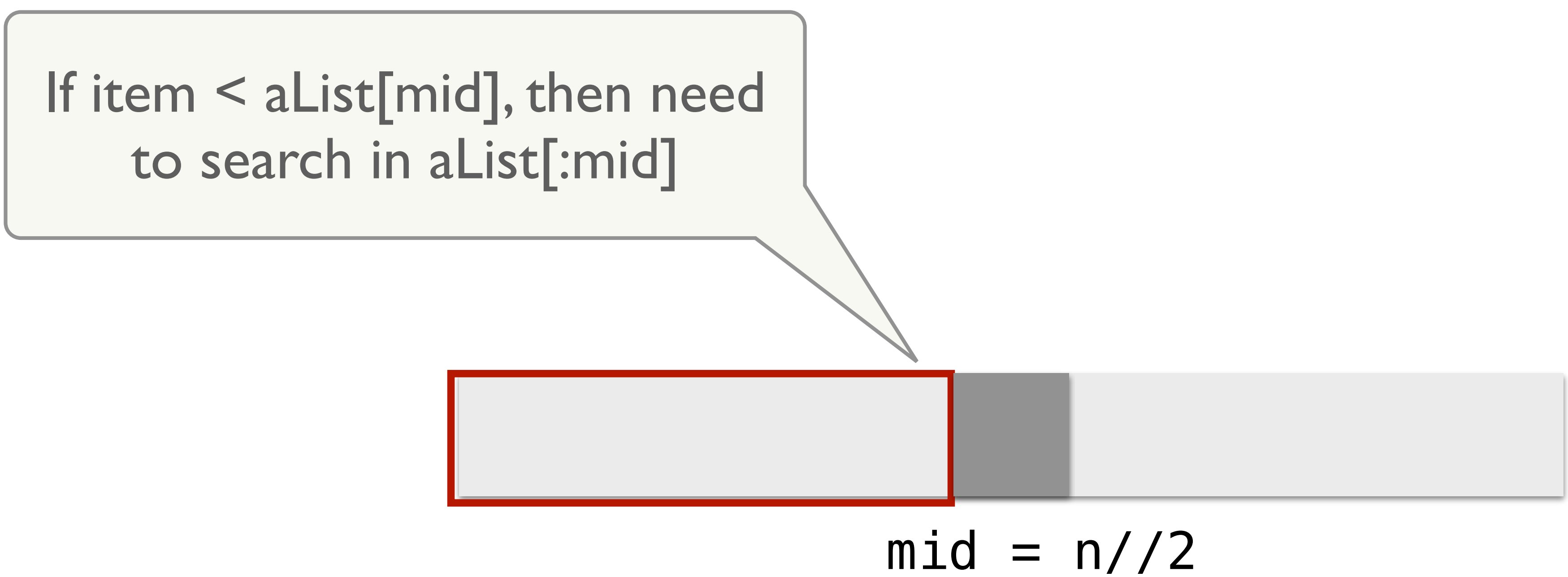
```
def binarySearch(aList, item):
    """Assume aList is sorted.
    If item is in aList, return True;
    else return False."""
    n = len(aList)
    mid = n // 2
    # base case 1
    if n == 0:
        return False

    # base case 2
    elif item == aList[mid]:
        return True
```



Binary Search

- Recursive case:
  - Recurse on left side if item is smaller than middle
  - Recurse on right side if item is larger than middle



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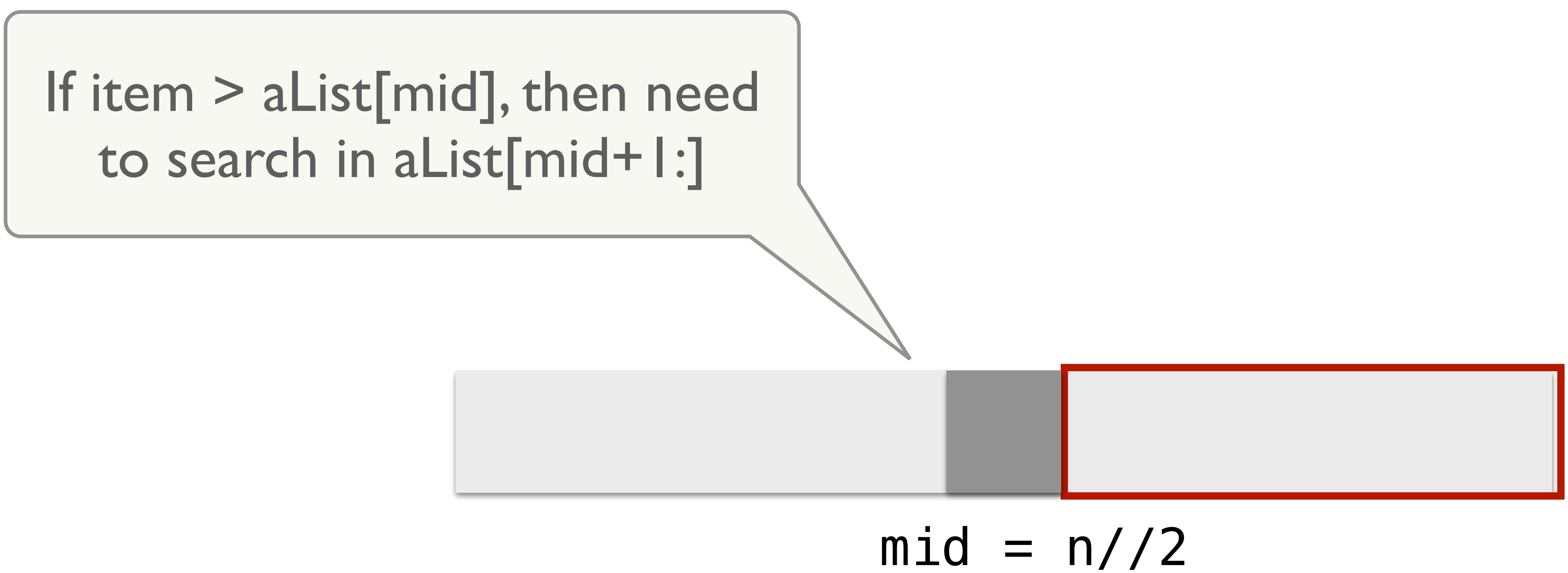

Binary Search

```
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    n = len(aList)
    mid = n // 2
    # base case 1
    if n == 0:
        return False

    # base case 2
    elif item == aList[mid]:
        return True

    # recurse on left
    elif item < aList[mid]:
        return binarySearch(aList[:mid], item)

    # recurse on right
    else:
        return binarySearch(aList[mid + 1:], item)
```