CS134 Lecture 19:
Recursion

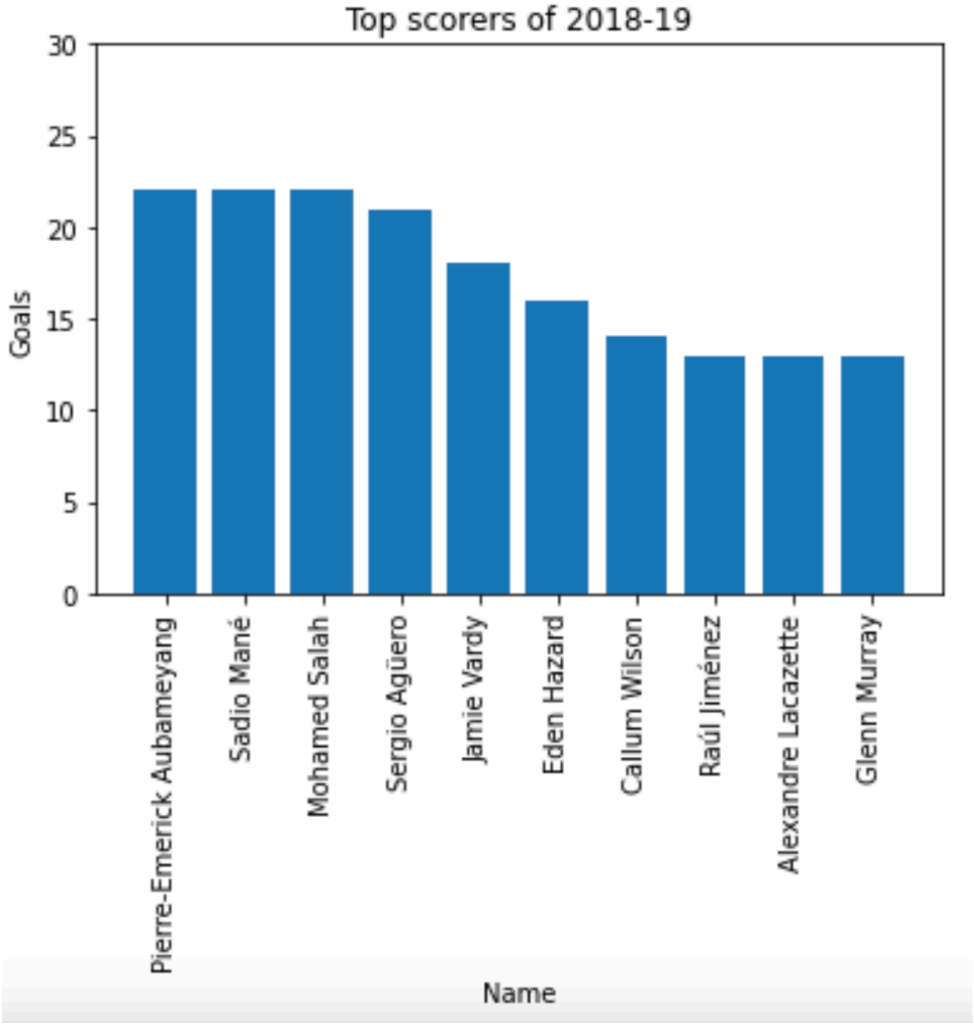
Announcements & Logistics

- **Lab 6 due Wed/Thurs at 10 pm**
  - Uses dictionaries, plotting, CSV files
- **HW 6** will be out today, due Mon at 10pm
- Lab 7, 8, and 9 are **partner labs**
  - Fill out google form sent by Lida by **noon tomorrow (Thursday)**!
  - Pair programming is an important skill as well as a vehicle for learning
- Pick up your **graded midterm exam** at the end of class
  - Will use last few mins of lecture to discuss the midterm

Do You Have Any Questions?

Last Time

- Worked through an example involving CSVs, dictionaries, and sets
- Discussed plotting with matplotlib
  - Python is pretty useful for data processing and visualization!



Today's Plan

Intro To Recursion

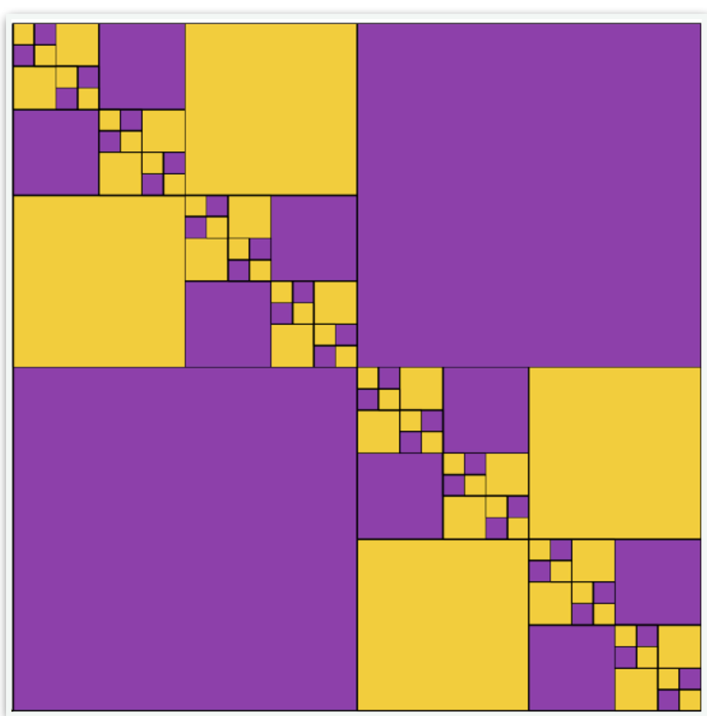
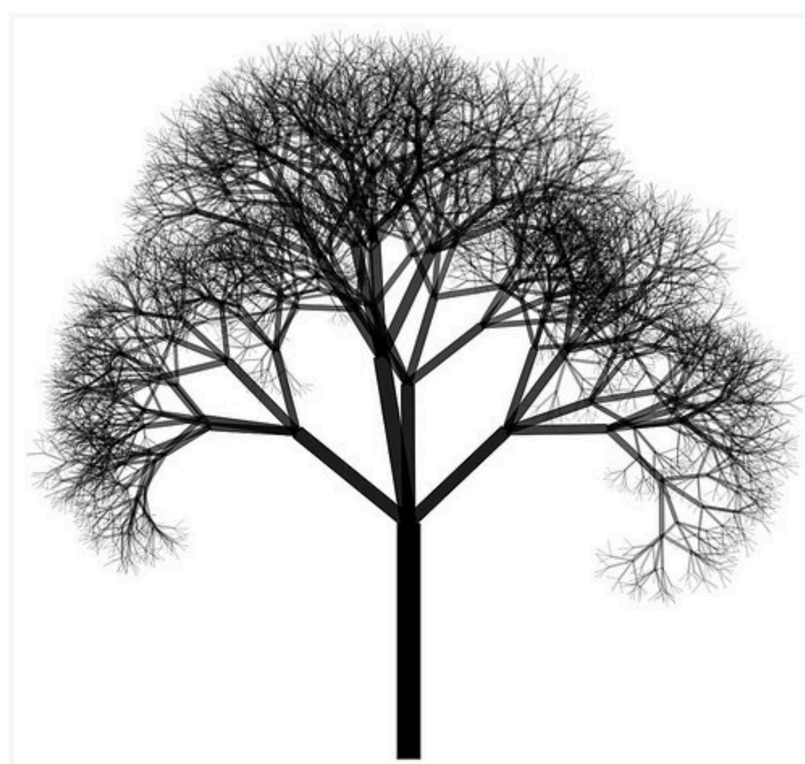
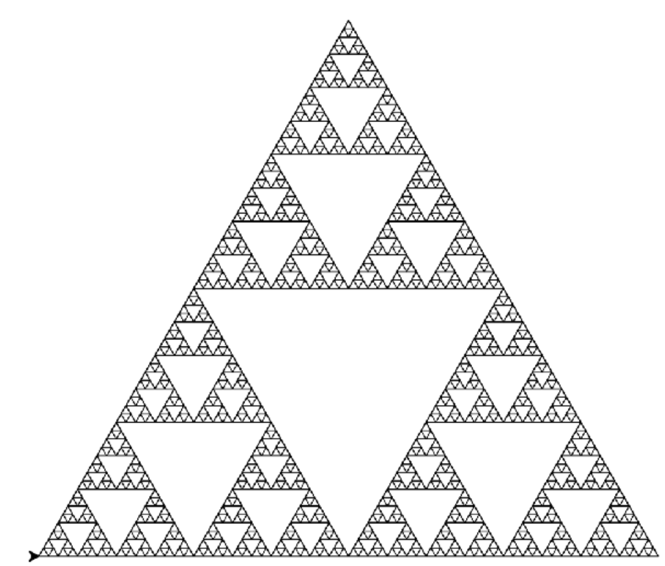
- Discuss what we mean by the term **recursion**
- Practice translating recursive ideas into recursive programs
- Examining the relationship between **recursive** and **iterative** programs
  - That is, how do recursive ideas relate to the iterative ideas (for loops, while loops) we've covered so far?

Where are We Going?

- First half of CS134: learned some **fundamental programming concepts**
    - Functions, conditionals, loops, data types
    - Built-in data structures and operations
- Looking ahead to the second half: more emphasis on **algorithmic** and **conceptual** topics: more "computational thinking"
    - **Recursion** (~1 week)
    - Defining our own **data types** using **classes and objects** (~2 weeks)
        - Object oriented programming topics
    - Continue developing our intuition regarding efficient vs inefficient code

Why Learn About Recursion?

- Recursion is an important problem solving paradigm
  - An alternative to **iteration** for repeatedly performing a task
  - Process that lets us "divide, conquer, combine"
  - Useful to build and maintain data structures (like trees and lists)
- Provides a different lens to view the world
  - If you love procrastination — recursion is just the thing for you!

So What Is Recursion?

- An alternative to **iteration** (loops) for repetition
- General problem solving idea:
  - Break the problem down to a smaller version of itself
  - Keep doing this until the problem is so small, the answer is straightforward
- Let's take an example of this approach
- **Example.** Write a function `count_down(n)` that prints integers `n, n-1,...,1` (one per line)
- How would we solve this using a loop?

Iterative: `count_down(n)`

- **Example.** Write a function `count_down(n)` that prints integers `n, n-1,..,1` (one per line)
- How would we solve this using a loop?

```
def count_down_iterative(n):
    '''Solution using loops'''
    for i in range(n):
        print(n - i)
```

Iterative: `count_down(n)`

- **Example.** Write a function `count_down(n)` that prints integers `n, n-1,..,1` (one per line)
- Now let's use recursion to do the same thing
- Recursion lets you solve this **without any loop**
    - Just using conditionals and functions

```
def count_down_iterative(n):
    '''Solution using loops'''
    for i in range(n):
        print(n - i)
```

Recursive: `count_down(n)`

- **Example.** Write a function `count_down(n)` that prints integers `n, n-1,..,1` (one per line)
- Key ideas to use recursion:
  - What's the smallest version of the problem we can *immediately* solve?
  - For larger versions of the problem, is there a small step we can take that brings us closer to the smallest version of the problem?

Recursive: `count_down(n)`

- **Example.** Write a function `count_down(n)` that prints integers `n, n-1,..,1` (one per line)
- Key ideas to use recursion:
  - What's the smallest version of the problem we can *immediately* solve?
    - `count_down(1)` just prints `1` and nothing else
  - For larger versions of the problem, is there a small step we can take that brings us closer to the smallest version of the problem?
    - to solve `count_down(n)`, printing `n` is the first step
    - the rest of the problem is the smaller version of the same problem!

Understanding Recursive Functions

- **Example.** Write a function `count_down(n)` that prints integers `n, n-1,..,1` (one per line)
- Recursive definition of countdown:
  - **Base case:** `n = 1`, `print(n)`
  - **Recursive rule:** `print(n), call count_down(n-1)`

Print and stop

Perform one step

Reduce the problem (or make the problem "smaller")

A function calling itself!

Recursive: `count_down(n)`

- **Example.** Write a function `count_down(n)` that prints integers `1, 2,...,n` (one per line)

```
def count_down(n):
    '''Prints numbers from n down to 1'''
    if n == 1:  # Base case
        print(n)
    else: # Recursive case: n > 1:
        print(n)
        count_down(n-1)
```

***Recursion:** A function calling itself!*

Understanding Recursive Functions

- Recursive functions seem to be able to reproduce looping behavior without writing any loops at all
- To understand what happens behind the scenes when a function calls itself, let's review what happens when a function calls another function
- Conceptually we understand function calls through the **function frame model**

Review:
Function Frame Model

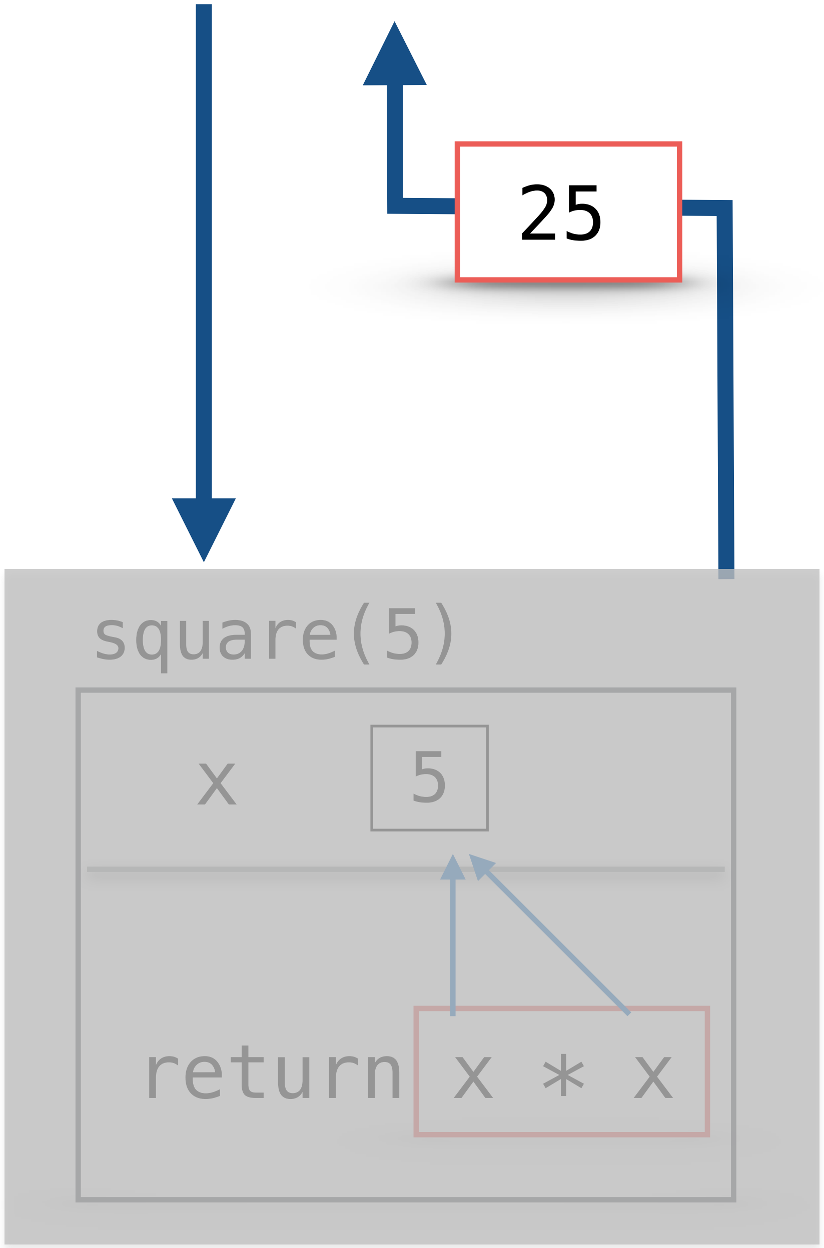
Review: Function Frame Model

- Consider a simple function **`square`**
- What happens when **`square(5)`** is invoked?

```
def square(x):
  return x*x
```

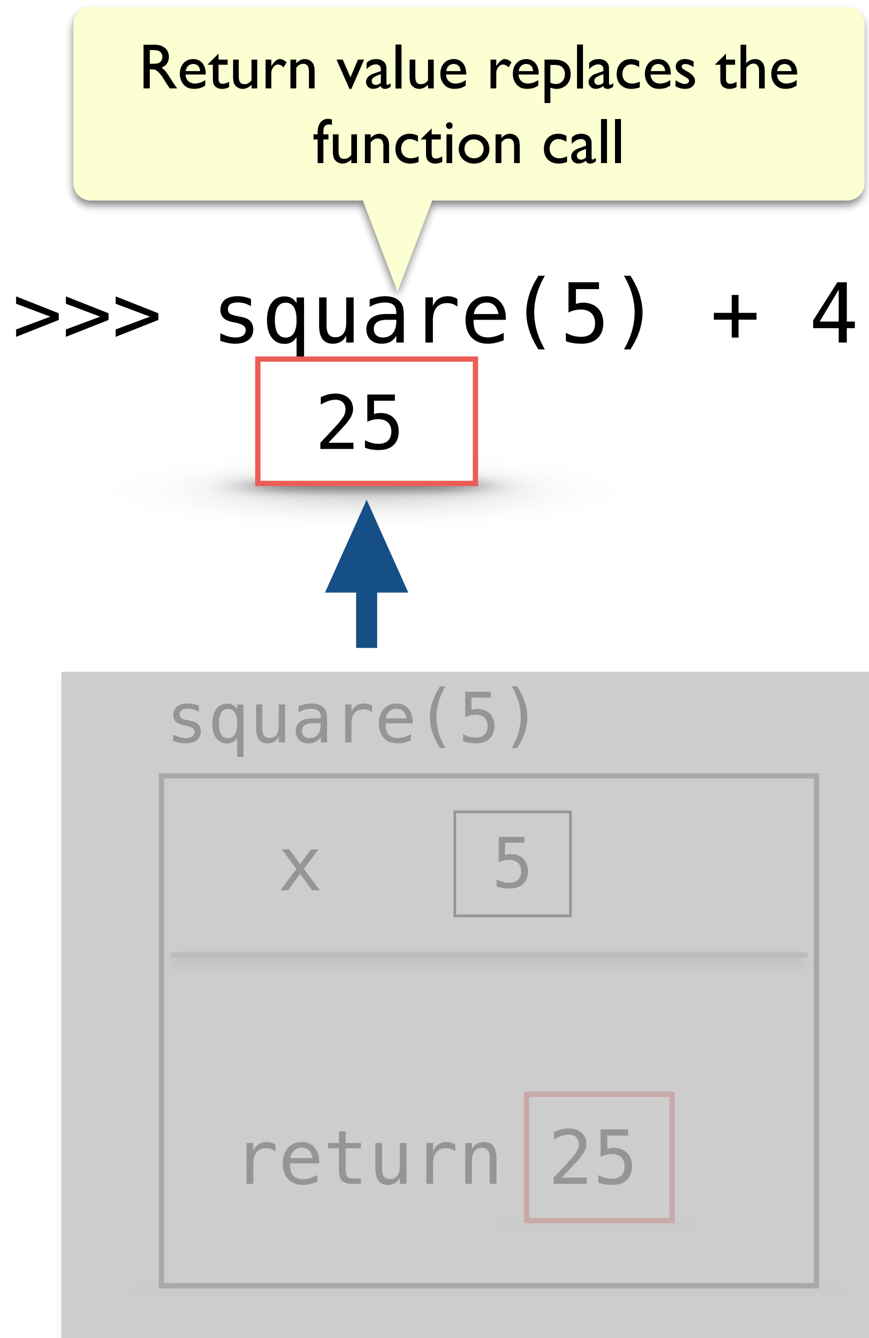
Review:
Function Frame Model

```
>>> square(5)
```



Summary:
Function Frame Model

- When we **return** from a function frame "control flow" goes back to where the function call was made
- Function frame (and the local variables inside it) **are destroyed after the return**
- If a function does not have an explicit return statement, it returns **None** after all statements in the body are executed



Review:
Function Frame Model

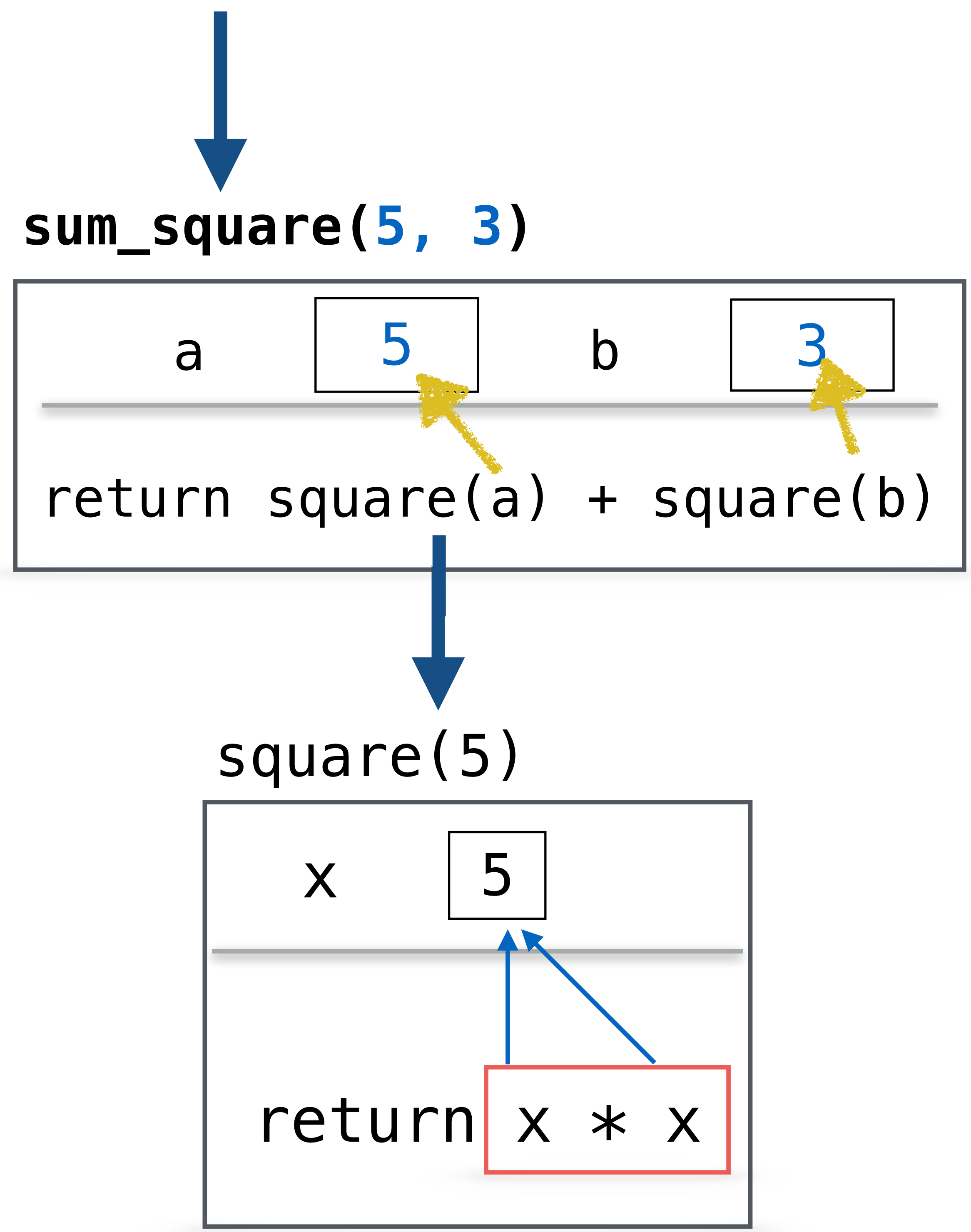
- How about functions that call other functions?

```
def sum_square(a, b):
  return square(a) + square(b)
```

- What happens when we call `sum_square(5, 3)`?

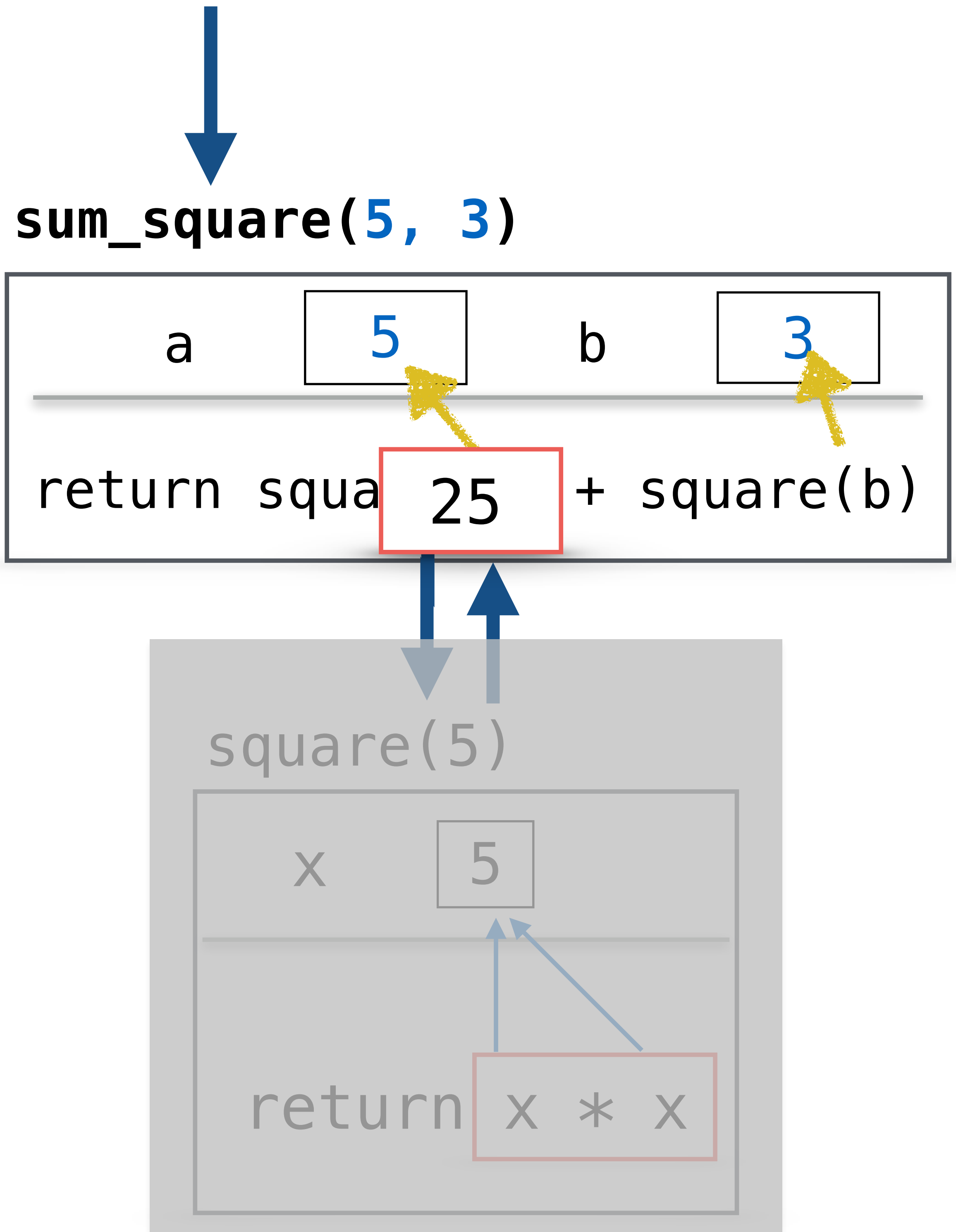
```
def sum_square(a, b):
  return square(a) + square(b)
```

```
>>> sum_square(5,3)
```



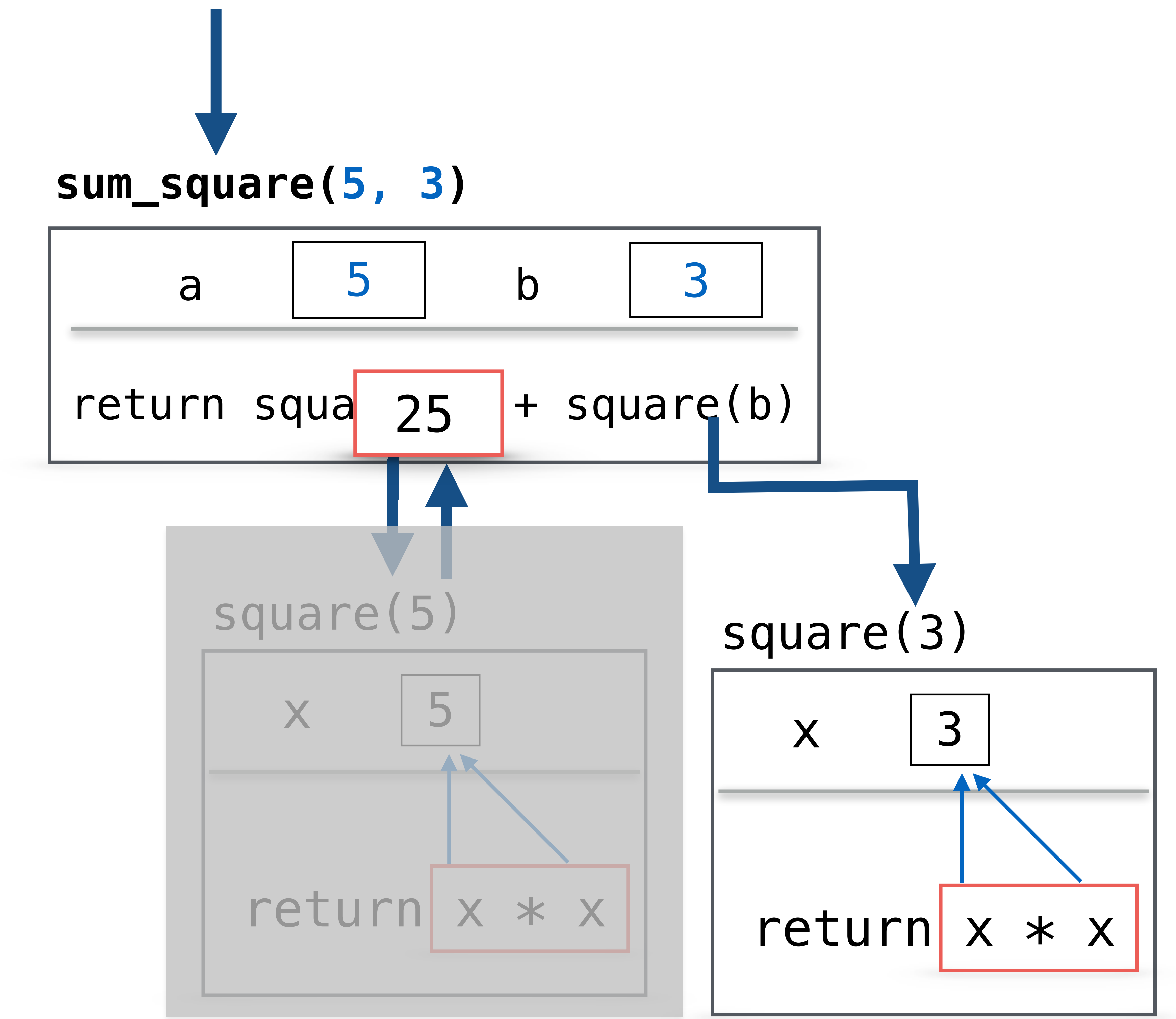
```
def sum_square(a, b):
   return square(a) + square(b)
```

```
>>> sum_square(5,3)
```



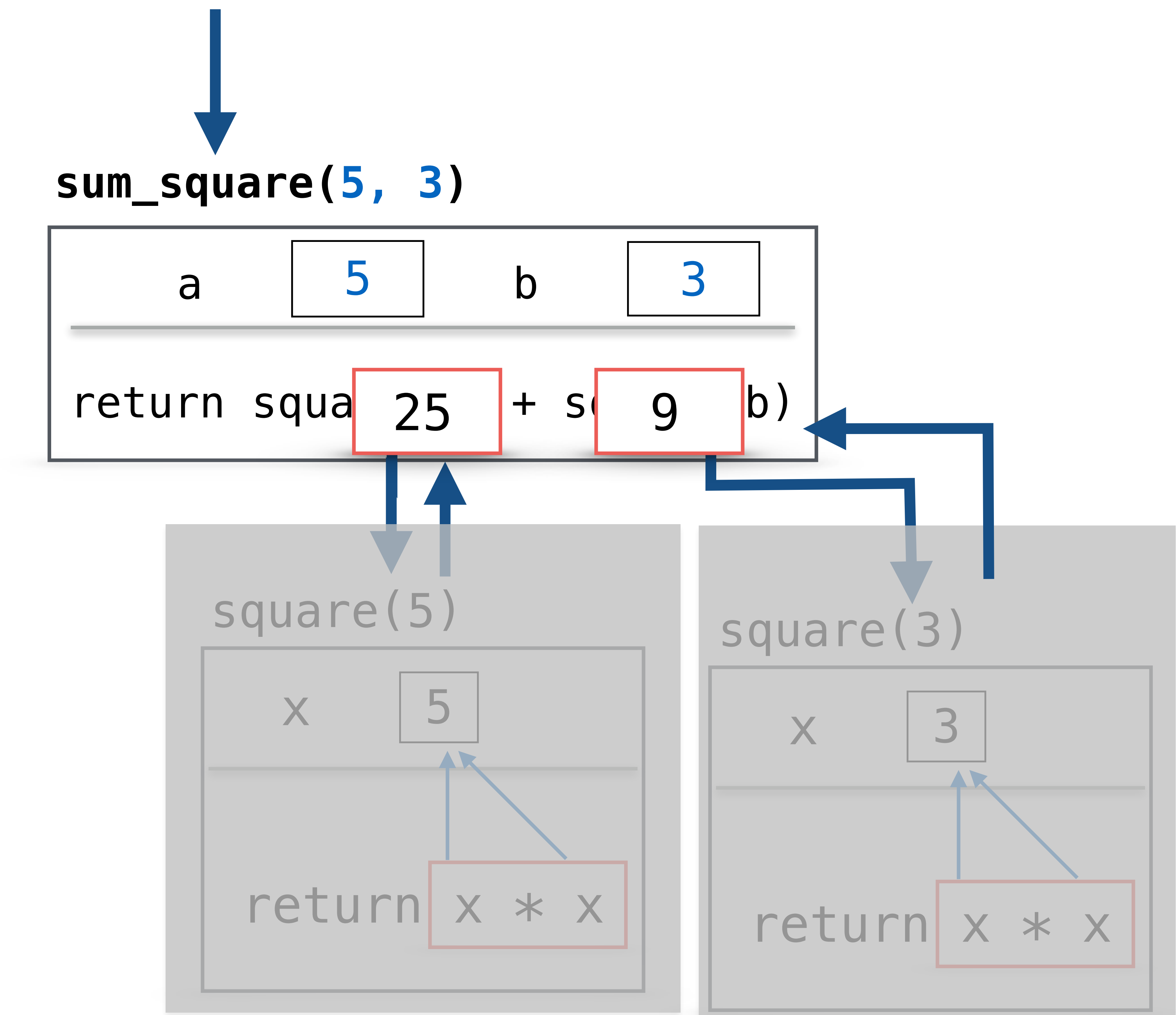
```
def sum_square(a, b):
   return square(a) + square(b)
```

```
>>> sum_square(5,3)
```

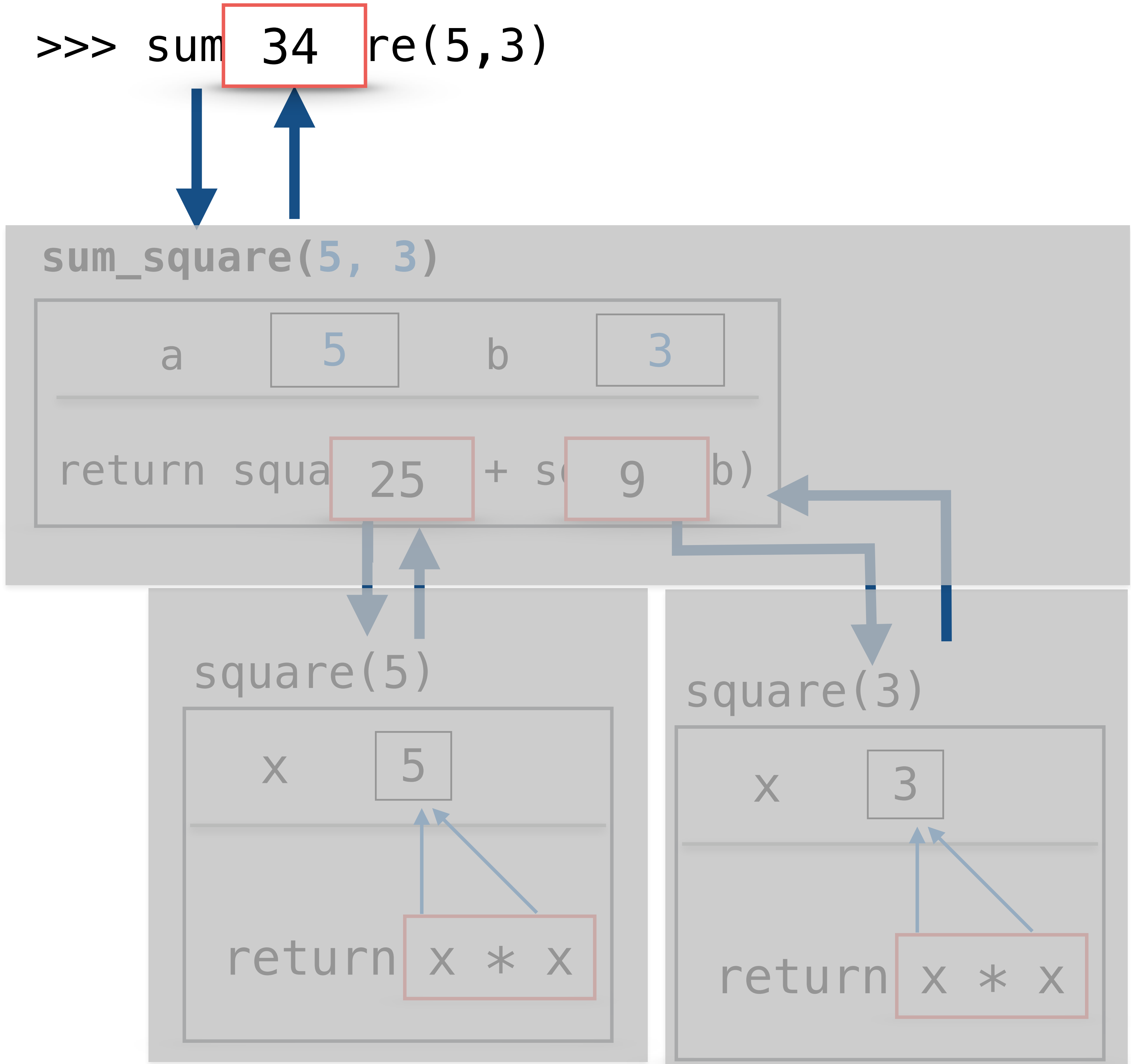


```
def sum_square(a, b):
   return square(a) + square(b)
```

```
>>> sum_square(5,3)
```



```
def sum_square(a, b):
   return square(a) + square(b)
```

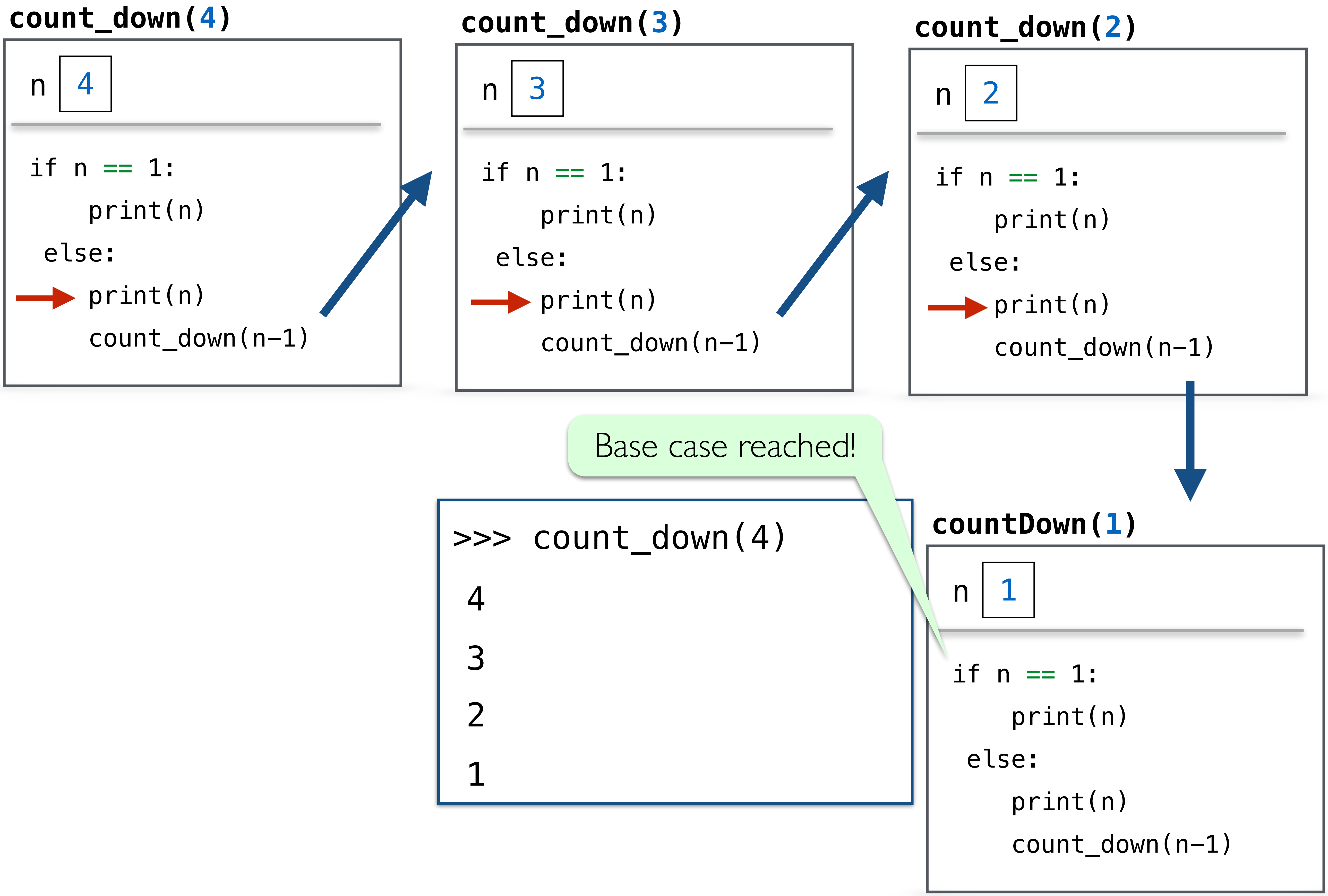


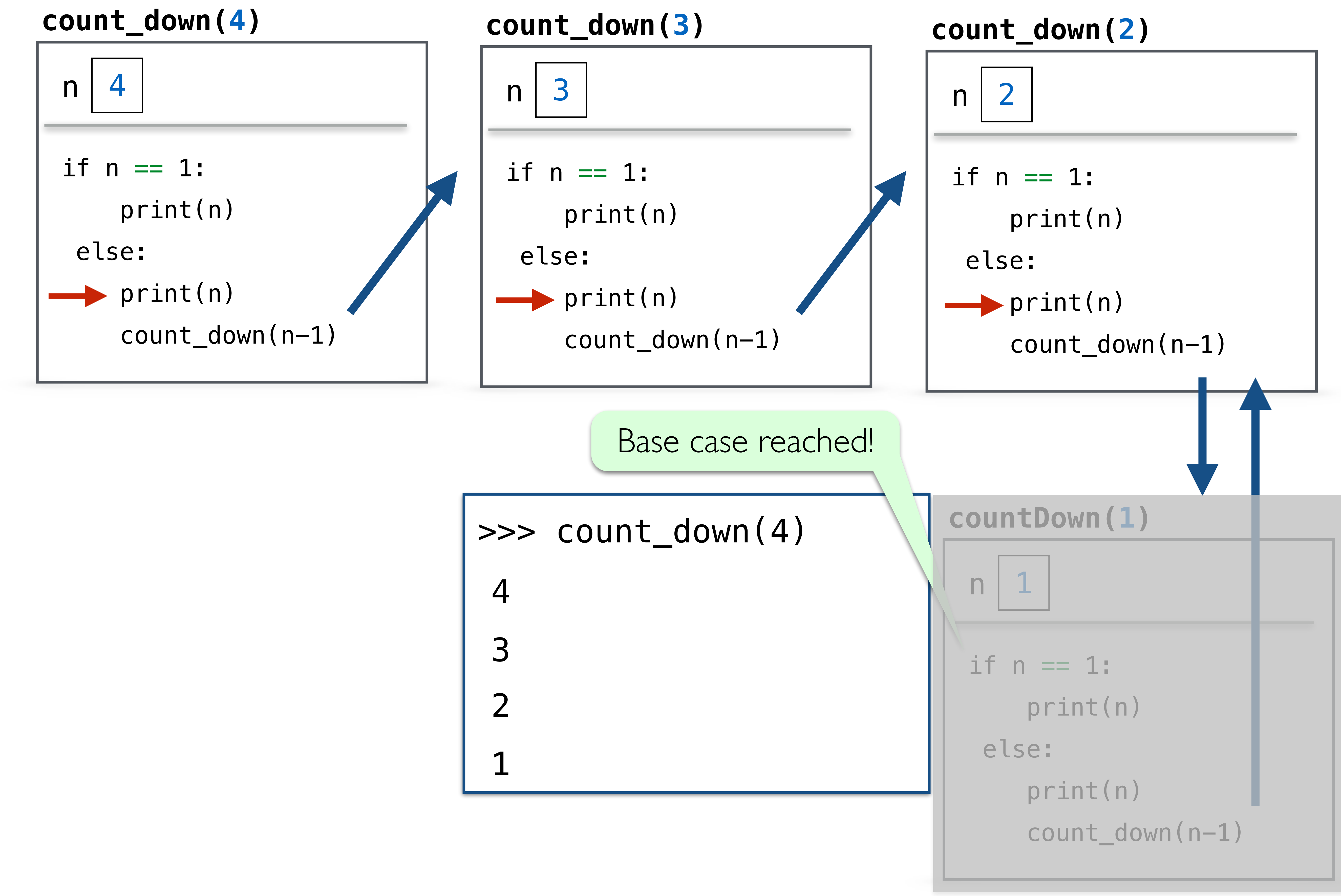
Function Frame Model to Understand `count_down`

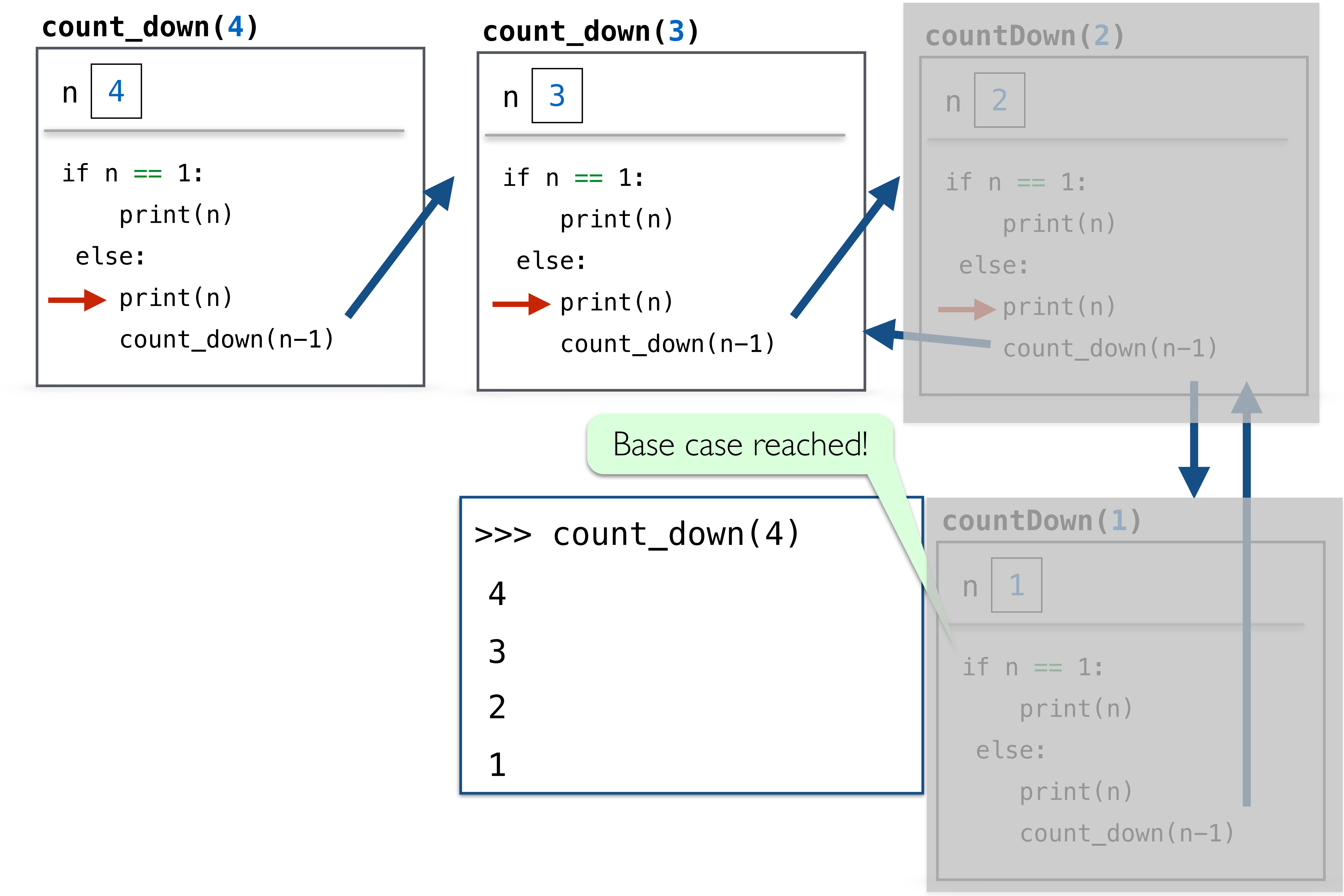
```python
def count_down(n):
    '''Prints ints from n down to 1'''
    if n == 1:
        print(n)
    else:
        print(n)
        count_down(n-1)
```

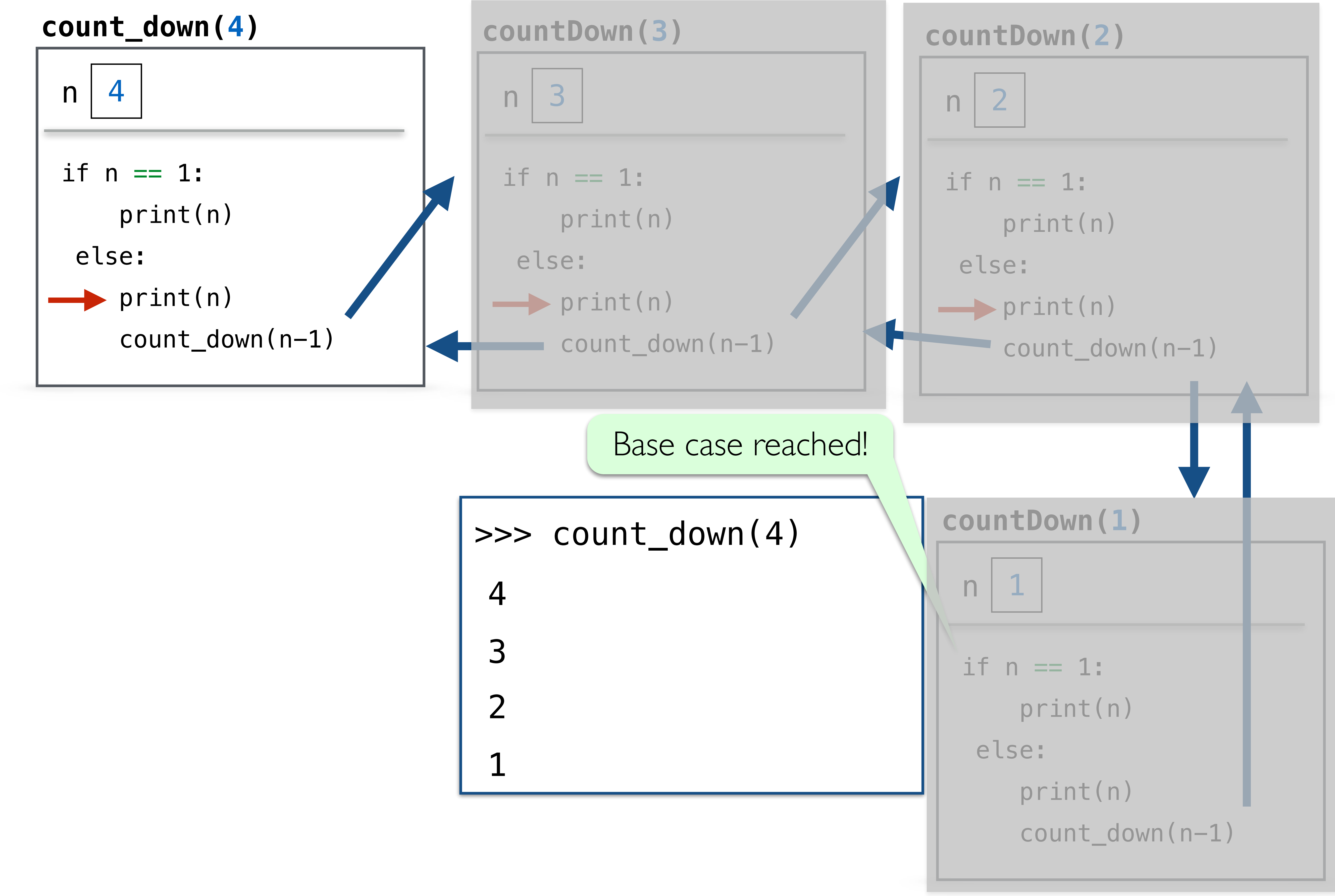
```
>>> val = count_down(5)
5
4
3
2
1
```

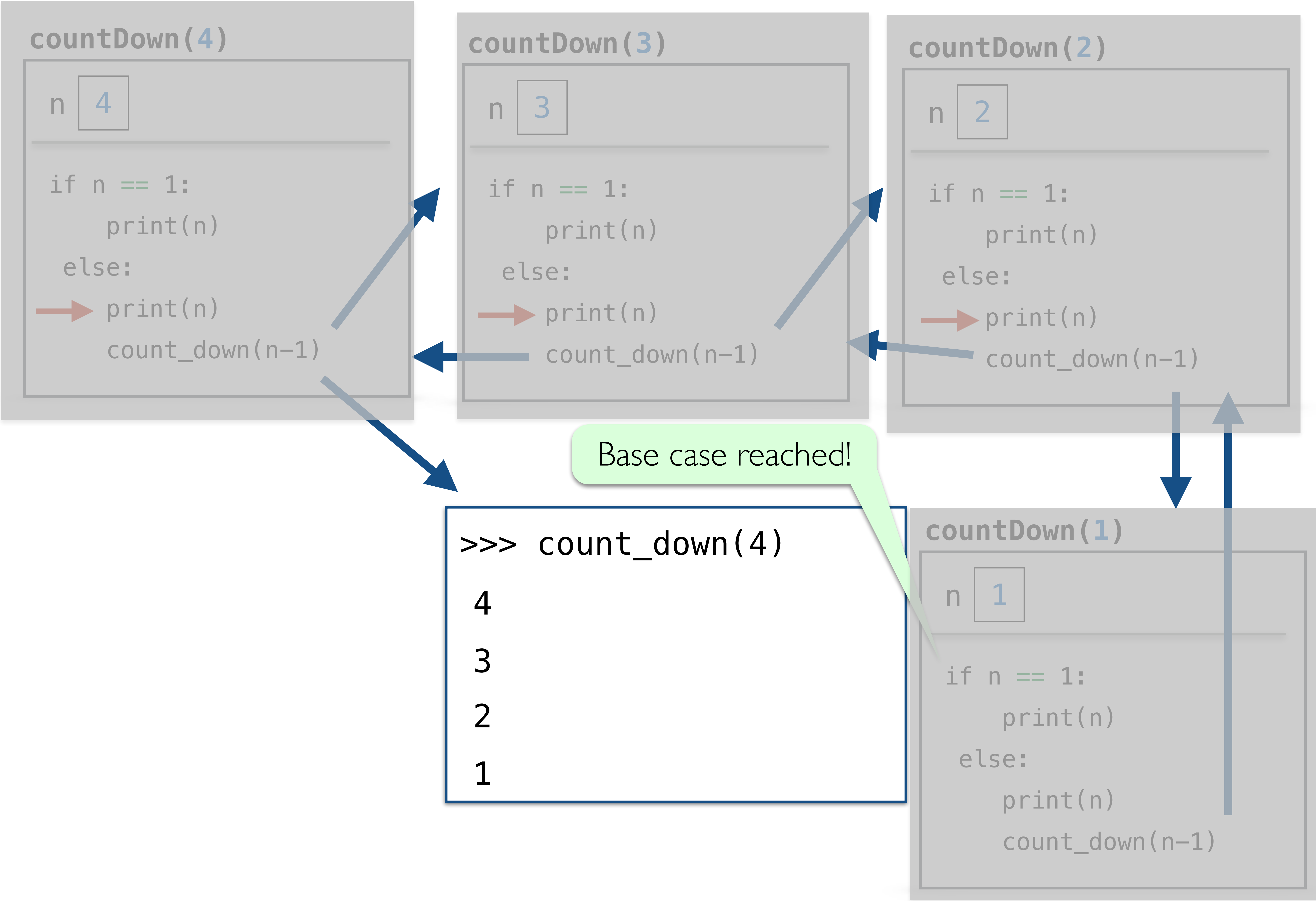
```
>>> val = count_down(4)
4
3
2
1
```


count_down(4)
n 4
if n == 1:
print(n)
else:
print(n)
count_down(n-1)
count_down(3)
n 3
if n == 1:
print(n)
else:
print(n)
count_down(n-1)
count_down(2)
n 2
if n == 1:
print(n)
else:
print(n)
count_down(n-1)
Base case reached!
>>> count_down(4)
4
3
2
1
countDown(1)
n 1
if n == 1:
print(n)
else:
print(n)
count_down(n-1)


count_down(4)
n 4
if n == 1:
print(n)
else:
print(n)
count_down(n-1)
count_down(3)
n 3
if n == 1:
print(n)
else:
print(n)
count_down(n-1)
count_down(2)
n 2
if n == 1:
print(n)
else:
print(n)
count_down(n-1)
Base case reached!
>>> count_down(4)
4
3
2
1
countDown(1)
n 1
if n == 1:
print(n)
else:
print(n)
count_down(n-1)


count_down(4)
n 4
if n == 1:
print(n)
else:
print(n)
count_down(n-1)
count_down(3)
n 3
if n == 1:
print(n)
else:
print(n)
count_down(n-1)
countDown(2)
n 2
if n == 1:
print(n)
else:
print(n)
count_down(n-1)
Base case reached!
>>> count_down(4)
4
3
2
1
countDown(1)
n 1
if n == 1:
print(n)
else:
print(n)
count_down(n-1)


count_down(4)
n 4
if n == 1:
print(n)
else:
print(n)
count_down(n-1)
countDown(3)
n 3
if n == 1:
print(n)
else:
print(n)
count_down(n-1)
countDown(2)
n 2
if n == 1:
print(n)
else:
print(n)
count_down(n-1)
Base case reached!
>>> count_down(4)
4
3
2
1
countDown(1)
n 1
if n == 1:
print(n)
else:
print(n)
count_down(n-1)


countDown(4)
n 4
if n == 1:
print(n)
else:
print(n)
count_down(n-1)
countDown(3)
n 3
if n == 1:
print(n)
else:
print(n)
count_down(n-1)
countDown(2)
n 2
if n == 1:
print(n)
else:
print(n)
count_down(n-1)
Base case reached!
>>> count_down(4)
4
3
2
1
countDown(1)
n 1
if n == 1:
print(n)
else:
print(n)
count_down(n-1)

TADA!

- Recursive functions may seem like magic at first glance, but they follow from the principles that we've been building all semester.
- It often takes several exposures to recursion before it "clicks", so we'll keep revisiting recursion in the coming lectures
  - Drawing pictures and practicing are two tools that can help
  - Our next lab is a partner lab so you can bounce your ideas off of a classmate and work though recursion stumbles

Recursive Approach to Problem Solving

- A recursive approach to problem solving has two main parts:
  - **Base case(s).** When the problem is **so small**, we solve it directly, without having to reduce it any further (this is when we stop)
  - **Recursive step.** Does the following things:
    - Performs an action that contributes to the solution (take one step)
    - **Reduces** the problem to a smaller version of the same problem, and calls the function on this **smaller subproblem** (break the problem down into a slightly smaller problem + one step)
- The recursive step is a form of "wishful thinking": assume the unfolding of the ***recursion*** will take care of the smaller problem by eventually reducing it to the base case
- In CS136/256, this form of wishful thinking will be introduced more formally as the *inductive hypothesis*

Counting with Recursion

- Recall the function `count_appearances(elem, l)`
  - Returns the number of times `elem` appears in `l`
- What the iterative way to implement this?

```
def count_occurrences(elem, l) :
    count = 0
    for item in l:
        if item == elem :
            count = count + 1
    return count
```

Examples today are easily written iteratively, but we'll be looking at problems on Friday where that may not be the case!

Recursive: `count_occurrences`

- One of the keys to thinking recursively:
  - What's the smallest version of the problem we can *immediately* solve?
  - For larger versions of the problem, is there a small step we can take that brings us closer to the smallest version of the problem?

```
def count_occurrences(elem, l) :
    '''recursive version'''
    # base case (empty list)
    if len(l) == 0:
        return 0
    else:
     # is first item same as elem?
     # if so, we can add 1
     # else, we add zero
     # now we have a smaller problem:
     # count # occurrences in smaller list
```

Recursive: count_occurrences

```
def count_occurrences(elem, l):
    '''recursive approach'''

    if len(l) == 0: # base case
        return 0

    else:  # recursive case
        first = 1 if elem == l[0] else 0
        rest = count_occurrences(elem, l[1:])

        return first + rest
```

Midterm Discussion

More Recursion:
`count_up`

count_up(n)

- Write a recursive function that prints integers from `1` up to `n`
- Recursive definition of countUp:
  - **Base case:** `n = 1, print(n)`
  - **Recursive rule:** `call count_up(n-1), print(n)`

We swapped the order of recursing (calling count_up) and printing

```
>>> count_up(5)

1
2
3
4
5
```

```
>>> count_up(4)

1
2
3
4
```

```
>>> count_up(3)

1
2
3
```

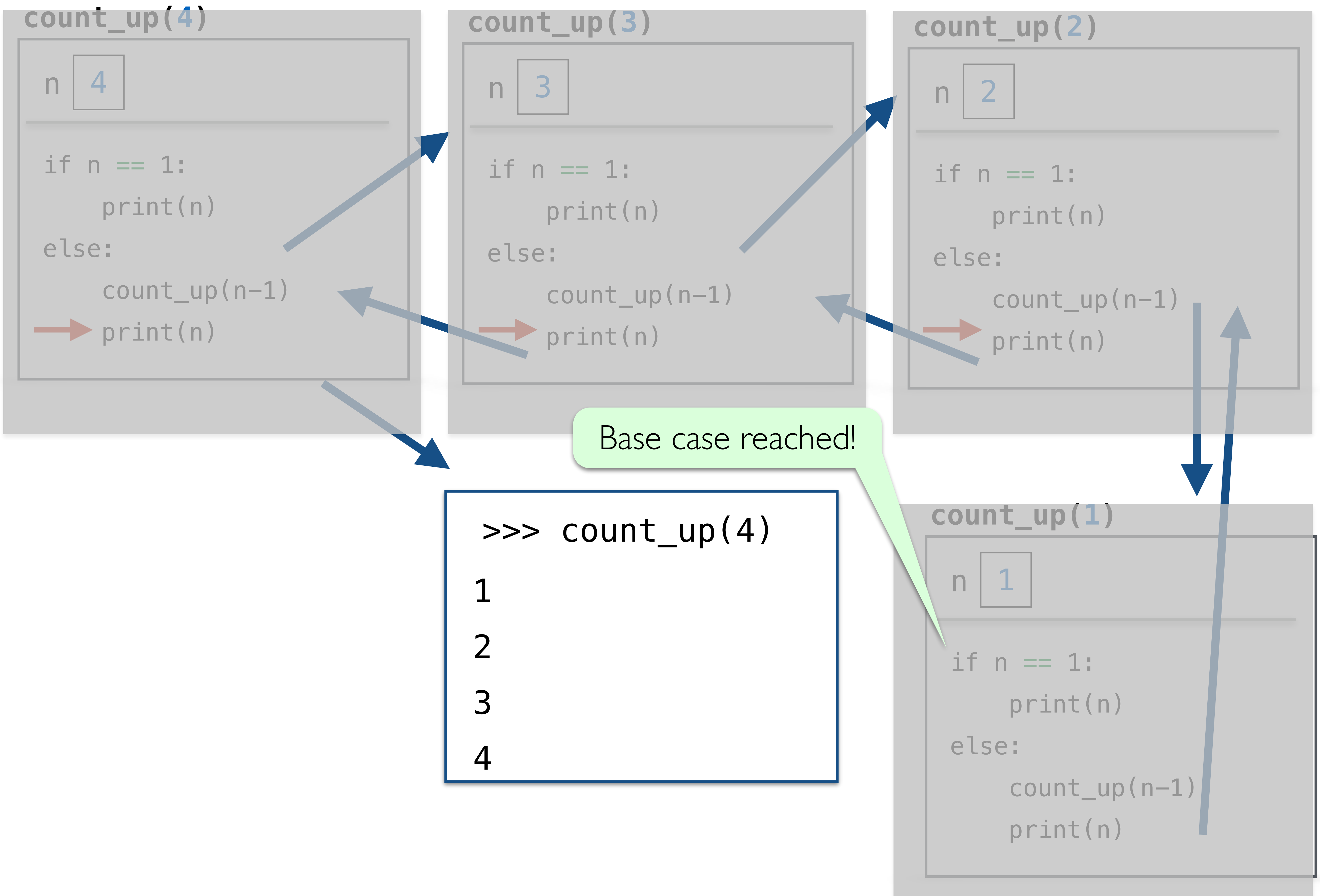
countUp(n)

- Note that unlike `count_down(n)` we moved our print statement to be **after** the recursive function call
- By printing **after** the recursive call, the print statement gets executed "on the way back" from recursive calls

```
def count_up(n):
    '''Prints out integers from 1 up to n'''
    if n == 1:
        print(n)
    else:
        count_up(n-1)
        print(n)
```

```
>>> count_up(5)
1
2
3
4
5
```

Function Frame Model to Understand `count_up`


count_up(4)
n 4
if n == 1:
print(n)
else:
count_up(n-1)
print(n)
count_up(3)
n 3
if n == 1:
print(n)
else:
count_up(n-1)
print(n)
count_up(2)
n 2
if n == 1:
print(n)
else:
count_up(n-1)
print(n)
Base case reached!
count_up(1)
n 1
if n == 1:
print(n)
else:
count_up(n-1)
print(n)
>>> count_up(4)
1
2
3
4

Recursion GOTCHAs!

GOTCHA #1

- If the problem that you are solving recursively **is not getting smaller**, that is, you are not getting closer to the base case --- **infinite recursion**!
- Never reaches the base case

```
def count_down_gotcha(n):
    '''Prints ints from 1 up to n’''
    if n == 1:  # Base case
        print(n)
    else:       # Recursive case
        print(n)
        count_down_gotcha(n)
```

Subproblem not getting smaller!

GOTCHA #2

- Missing base case/unreachable base case--- another way to cause **infinite recursion**!

```
def print_halves_gotcha(n):
    """Prints n, n/2, down to ... 1"""
    if n > 0:
        print(n)
        return print_halves_gotcha(n/2)
```

Always true!

"Maximum recursion depth exceeded"

- In practice, the infinite recursion examples will terminate when Python runs out of resources for creating function call frames, leads to a "maximum recursion depth exceeded" error message

Next Lectures

- Intro to `turtle` module and graphical recursion
- Comparing iterative and recursive programs