Announcements

- Look for homework 5 online
- Grading plans:
  - Midterm
  - Lab 5
  - Homework 4
Today’s Plan

- Analysis of Ethernet Efficiency
- A Bit on Switching
Summary of Ethernet

- Carrier Sense = Wait if network idle
- 1-persistence = If waiting, start when idle
- Collision Detection = Stop and Backoff
- Minimum packet transmission time = $2 \times \text{max propagation time} = \text{waiting slot time}$
- Backoff = Delay random # between 0 and $2^{\text{failures}} - 1$ slots after collision
Efficiency = \frac{P/R}{W \times T + P/R}

W = \frac{1 - A}{A}

A = \left(1 - \frac{1}{Q}\right)(Q - 1)

P = \text{expected/average packet size}
R = \text{transmission rate (M & B call it C)}
W = \text{expected \# of slots between transmissions}
T = \text{expected length of a contention slot}
A = \text{Probability exactly one computer sends in a slot}
Q = \text{number of computers trying to sent}
Efficiency = \frac{P/R}{W \times T + P/R}

= \frac{1}{\frac{W \times T + 1}{P/R}}

P = \text{expected/average packet size}
R = \text{transmission rate (M \& B call it C)}
W = \text{expected \# of slots between transmissions}
T = \text{expected length of a contention slot}
Probability Principles

Principle 1: The probability that something won’t happen is 1 minus the probability that it will happen.
$A$ = Probability that exactly 1 computer attempts a transmission in a given slot

$W$ = expected # of slots between transmissions

$W = \frac{1 - A}{A} = \frac{\text{Prob(0 or > 1 transmits)}}{\text{Prob(exactly 1 transmits)}}$
Expected spins between consecutive 7’s?
Expected spins between consecutive multiples of 5?
$\text{Prob}(\# \text{ picked is multiple of 5}) = \frac{1}{5}$

$\text{Prob}(\# \text{ picked is not mult. of 5}) = \frac{4}{5}$

Expected spins between mult. of 5 = 4
A = Probability that exactly 1 computer attempts a transmission in a given slot

W = expected # of slots between transmissions

\[
W = \frac{1 - A}{A} = \frac{\text{Prob( 0 or > 1 transmits)}}{\text{Prob(exactly 1 transmits)}}
\]
$W = \text{Expected \# of slots wasted}$
Great Expectations

Principle 2: Computing Expected Values:

Given F: outcome $\rightarrow$ value

$$\text{Expected}(F) = \sum \text{Prob}(\text{outcome}) \times F(\text{outcome})$$

all outcomes
Expected Contention Slot

Outcomes: \{ X \text{ any } Y \text{ collide then } X \text{ delays } 0 \text{ and transmits, } X \text{ and } Y \text{ collide then collide again then } Y \text{ delays } 0 \text{ and transmits, } \ldots \} \\
\sum F( \text{collide, } X \text{ delays } 0 \& \text{ transmits } ) = 1 \text{ (slots wasted)}

W = \text{Expected } \# \text{ of slots wasted} = \sum_{\text{all } i} \text{Prob( 1st success in slot } i \text{ ) } \times i
Efficiency = \frac{P/R}{W \times T + P/R}

W = \frac{1 - A}{A}

A = (1 - \frac{1}{Q})(Q - 1)

P = \text{expected/average packet size}

R = \text{transmission rate (M & B call it C)}

W = \text{expected \# of slots between transmissions}

T = \text{expected length of a contention slot}

A = \text{Probability exactly one computer sends in a slot}

Q = \text{number of computers trying to send}
Probability a particular computer tries to send $\approx \frac{1}{S}$

$S = \text{The number of backoff slots the computer is currently using}$
S = The number of backoff slots all of the computers are currently using!!!

Q = The total number of computers that are trying to send

Probability a particular computer sends alone ≈ ???
Probability Principles

Principle 3: The probability of two independent events both happening is the product of their separate probabilities if the events happen (or don’t happen) independently.
S = The number of backoff slots all of the computer are currently using!!!

Q = The total number of computers that are trying to send

Probability a particular computer sends alone

\[
\approx \frac{1}{S} \left( 1 - \frac{1}{S} \right)^{Q-1}
\]

S = The number of backoff slots all of the computer are currently using!!!

Q = The total number of computers that are trying to send
Probability of Success

Probability some lucky computer sends alone

\[ \approx \frac{Q}{S} \left( 1 - \frac{1}{S} \right) (Q - 1) \]

\( S \) = The number of backoff slots all of the computer all currently using
\( Q \) = The total number of computers that are trying to send
Probability of Success

$$A \approx \frac{Q}{S}(1 - \frac{1}{S})(Q - 1)$$

$S =$ The number of backoff slots all of the computer all currently using
$Q =$ The total number of computers that are trying to send
But Metcalfe and Boggs Say...

\[ A = \left( 1 - \frac{1}{Q} \right)(Q - 1) \]

\[ S = \text{The number of backoff slots all of the computer all currently using} \]
\[ Q = \text{The total number of computers that are trying to send} \]
We assume that a queued station attempts to transmit in the current slot with probability \(1/Q\), or delays with probability \(1-(1/Q)\); this is known to be the optimum statistical decision rule, approximated in Ethernet stations by means of our load-estimating retransmission control algorithms [20, 21].

6.1 Acquisition Probability

We now compute \(A\), the probability that exactly one station attempts a transmission in a slot and therefore acquires the Ether. \(A\) is \(Q \times (1/Q) \times ((1 - (1/Q))^{(Q-1)})\); there are \(Q\) ways in which one station can choose to transmit (with probability \((1/Q)\)) while \(Q-1\) stations choose to wait (with probability \(1 - (1/Q)\)). Simplifying,
We assume that a queued station attempts to transmit in the current slot with probability \(1/Q\), or delays with probability \(1 - (1/Q)\); this is known to be the optimum statistical decision rule, approximated in Ethernet stations by means of our load-estimating retransmission control algorithms [20, 21].

6.1 Acquisition Probability

We now compute \(A\), the probability that exactly one station attempts a transmission in a slot and therefore acquires the Ether. \(A\) is \(Q \cdot (1/Q) \cdot ((1 - (1/Q))^{Q-1})\); there are \(Q\) ways in which one station can choose to transmit (with probability \((1/Q)\)) while \(Q - 1\) stations choose to wait (with probability \(1 - (1/Q)\)). Simplifying,
Thinking Optimistically

\[ A \approx \frac{Q}{S} \left( 1 - \frac{1}{S} \right)^{Q-1} \]

\[ \frac{dA}{dS} \approx \frac{Q}{S^3} (Q - S) \left( 1 - \frac{1}{S} \right)^{(Q-2)} \]

\( S = \text{The number of backoff slots all of the computer all currently using} \)

\( Q = \text{The total number of computers that are trying to send} \)
Probability of Success?

\[ A \approx \frac{Q}{S} \left( 1 - \frac{1}{S} \right)^{(Q - 1)} \]

\[ < \left( 1 - \frac{1}{Q} \right)^{(Q - 1)} \]

- \( S = \) The number of backoff slots all of the computer are currently using
- \( Q = \) The total number of computers that are trying to send
Efficiency = \frac{P/R}{W \times T + P/R}

= \frac{1}{W \times T + 1}

P = \text{expected/average packet size}
R = \text{transmission rate (M \& B call it C)}
W = \text{expected \# of slots between transmissions}
T = \text{expected length of a contention slot}
## The Bottom Line

<table>
<thead>
<tr>
<th>N = Q</th>
<th>P = 4K</th>
<th>P = 1K</th>
<th>P = 512</th>
<th>P = 48</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0.99</td>
<td>0.94</td>
<td>0.89</td>
<td>0.44</td>
</tr>
<tr>
<td>5</td>
<td>0.98</td>
<td>0.94</td>
<td>0.88</td>
<td>0.41</td>
</tr>
<tr>
<td>10</td>
<td>0.98</td>
<td>0.93</td>
<td>0.87</td>
<td>0.39</td>
</tr>
<tr>
<td>128</td>
<td>0.98</td>
<td>0.93</td>
<td>0.86</td>
<td>0.37</td>
</tr>
</tbody>
</table>
MEASURED CAPACITY OF AN ETHERNET

SIGCOMM '88 Symposium proceedings on Communications architectures and protocols

David R. Boggs
Jeffrey C. Mogul
Christopher A. Kent
Efficiency = \frac{P/R}{W \times T + P/R}

= \frac{1}{W \times T + 1}

\frac{1}{P/R}

P = \text{expected/average packet size}

R = \text{transmission rate (M \& B call it C)}

W = \text{expected \# of slots between transmissions}

T = \text{expected length of a contention slot}