# CS134 Lecture 20: More Recursion

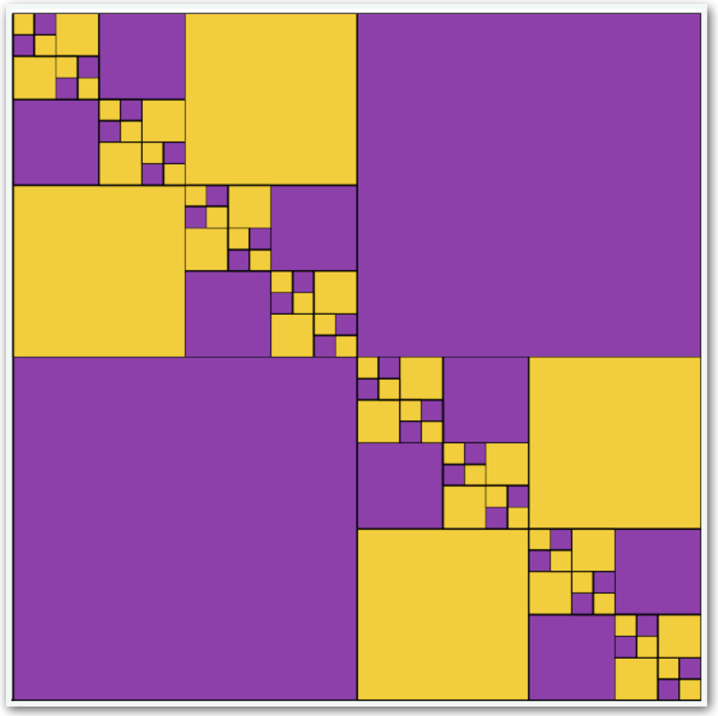
# Announcements & Logistics

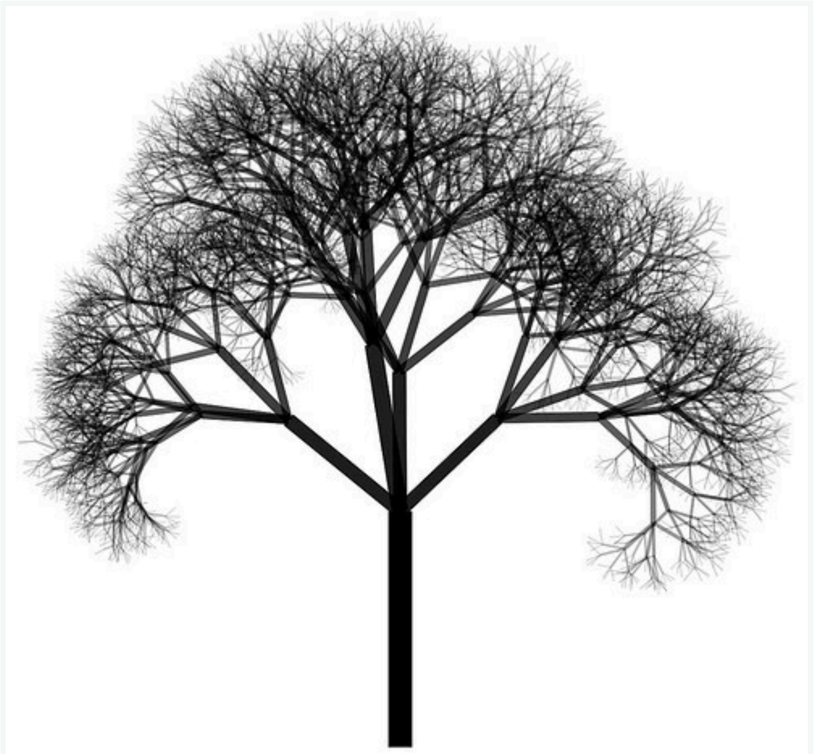
- **HW 6** on GLOW due Mon at 10pm
    - Good practice for short-code questions on exam
    - Practice on pencil and paper first
- Lab 7, 8, and 9 are **partner labs**
    - Pair programming is an important skill as well as a vehicle for learning
- Colloquium Today: Tim Randolph '18
    - Theoretical computer science talk on the Subset Sum problem (a problem you may use a "brute-force" approach to solve recursively in a future assignment!)

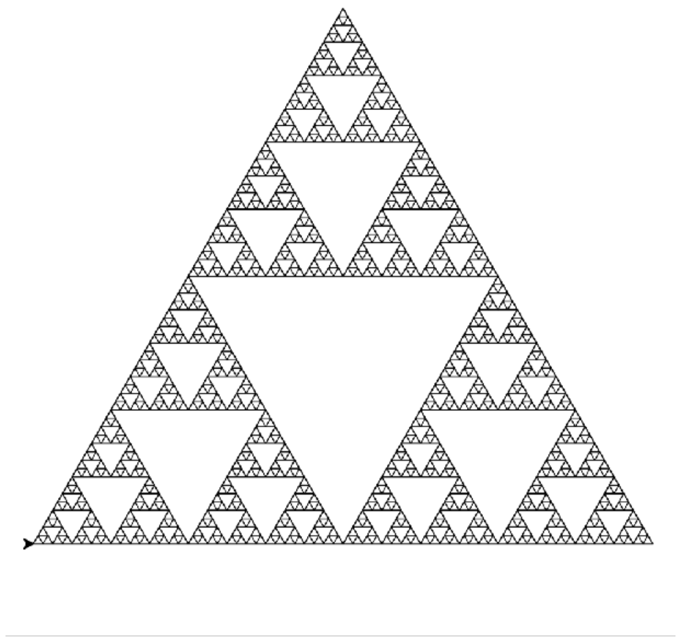
**Do You Have Any Questions?**

# Last Time

- Introduction to recursion
  - Recursion as a new problem solving paradigm
  - Recursion as an alternative to iteration
  - Recursive solution to a familiar problem (count elements in a list)







# Last Time: Recursive Approach to Problem Solving

- A recursive function is a function **that calls itself**
- A recursive approach to problem solving has two main parts:
  - **Base case(s).** When the problem is **so small**, we can solve it directly, without having to reduce it any further
  - **Recursive step.** Does the following things:
    - Performs an action that contributes to the solution
    - **Reduces** the problem to a smaller version of the same problem, and calls the function on this **smaller subproblem**
- The recursive step is a form of "wishful thinking" (also called the inductive hypothesis)

# Understanding Recursive Functions

- Let's review a simple recursive function that gives us some intermediate feedback through `print` statements:
  - we'll write a recursive function to print integers from `n` down to `1`
- Recursive definition of countdown:
  - **Base case:** `n = 1`, `print(n)` — Print and stop
  - **Recursive rule:** `print(n), call count_down(n-1)`
    - `print(n)`: Perform one step
    - `call count_down(n-1)`: Reduce the problem (or make the problem "smaller")

# Understanding Recursive Functions

- Recursive definition of countdown:
  - Base case: `n = 1, print(n)`
  - Recursive rule: `print(n), count_down(n-1)`

```
def count_down(n):
    '''Prints numbers from n down to 1'''
    # Base case: n == 1
    if n == 1:
        print(n)
    # Recursive case: n > 1:
    else:
        print(n)
        count_down(n-1)
```

```
>>> result = count_down(5)
5
4
3
2
1
```

# Understanding Recursive Functions

- Recursive functions seem to be able to reproduce looping behavior without writing any loops at all
- To understand what happens behind the scenes when a function calls itself, let's review what happens when a function calls another function
- Conceptually we understand function calls through the **function frame model**

# Review:
# Function Frame Model

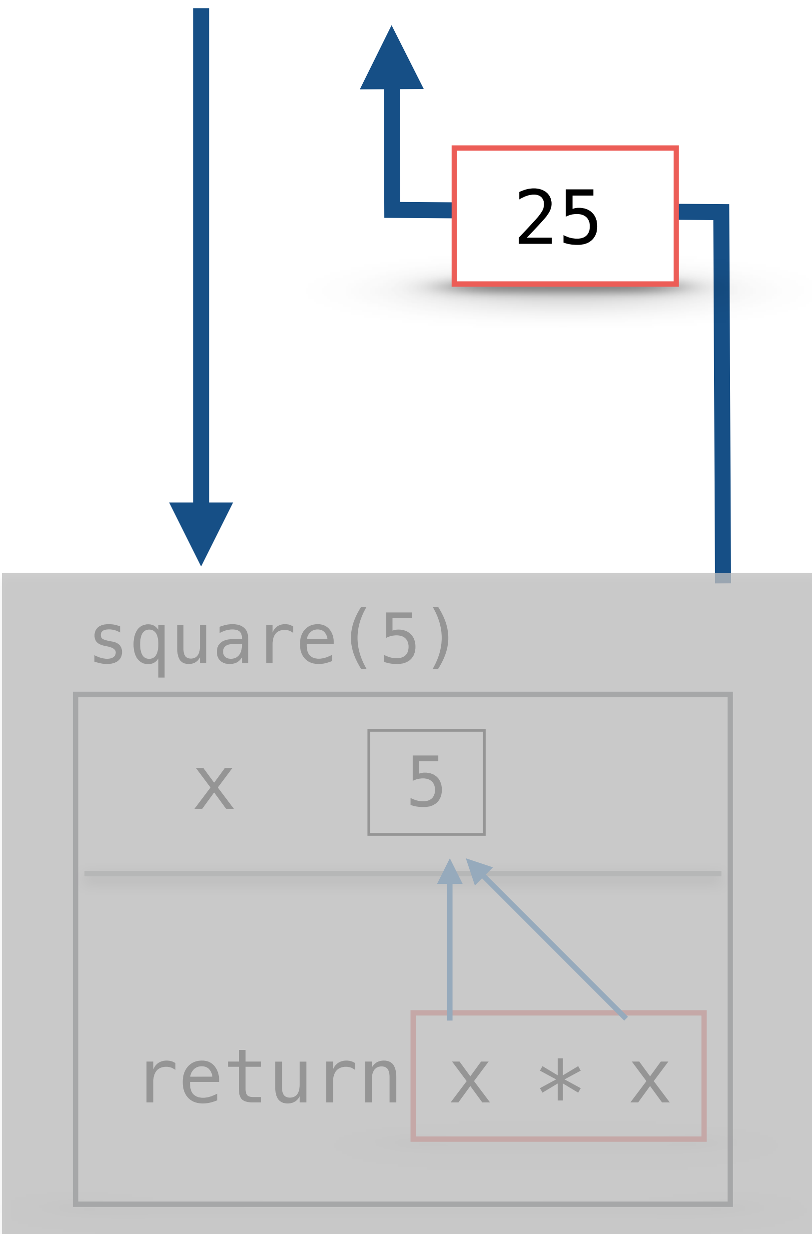
# Review: Function Frame Model

- Consider a simple function **`square`**
- What happens when **`square(5)`** is invoked?

```
def square(x):
  return x*x
```

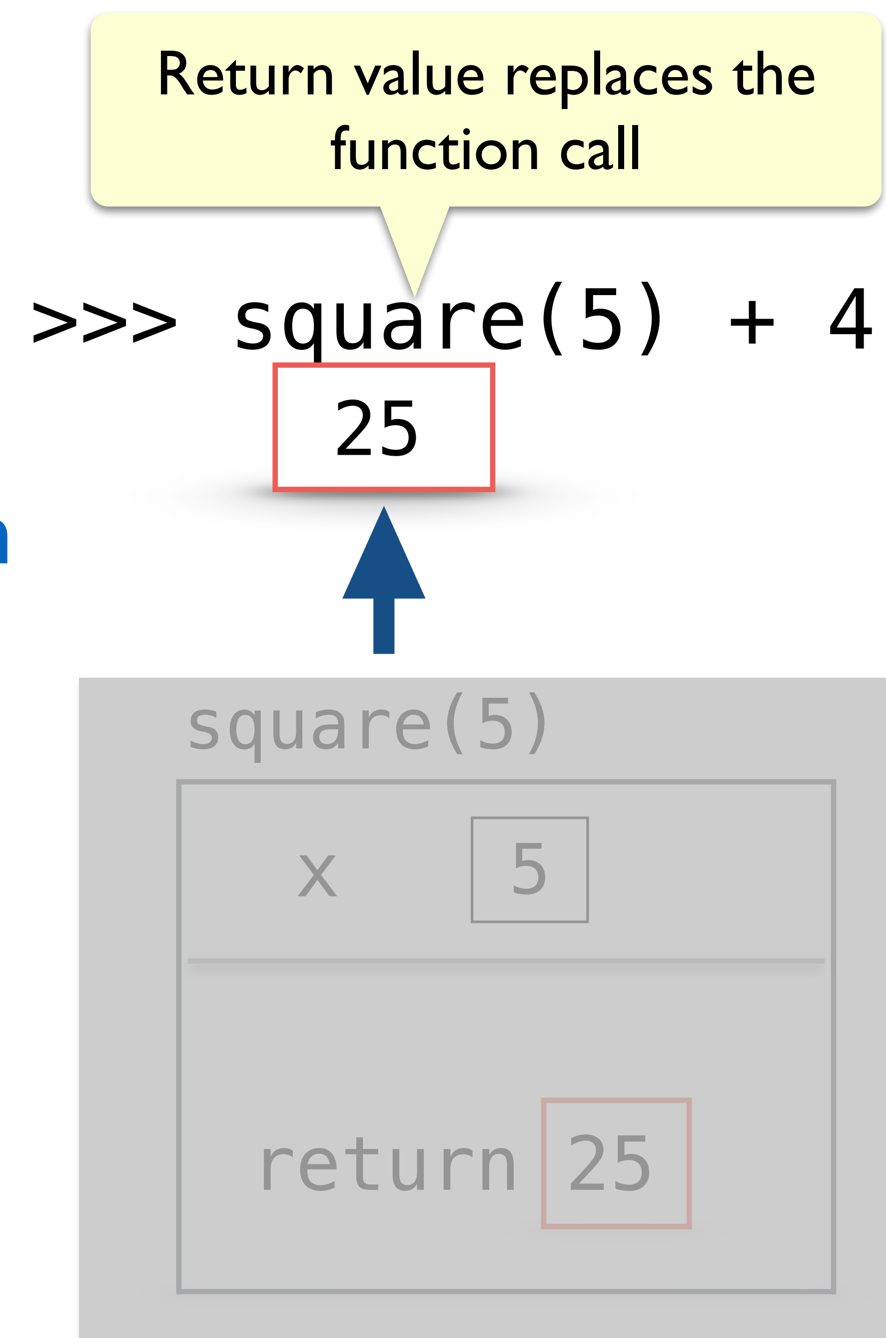
# Review:
# Function Frame Model

```
>>> square(5)
```



# Summary:
# Function Frame Model

- When we **return** from a function frame "control flow" goes back to where the function call was made
- Function frame (and the local variables inside it) **are destroyed after the return**
- If a function does not have an explicit return statement, it returns **None** after all statements in the body are executed



# Review:
# Function Frame Model

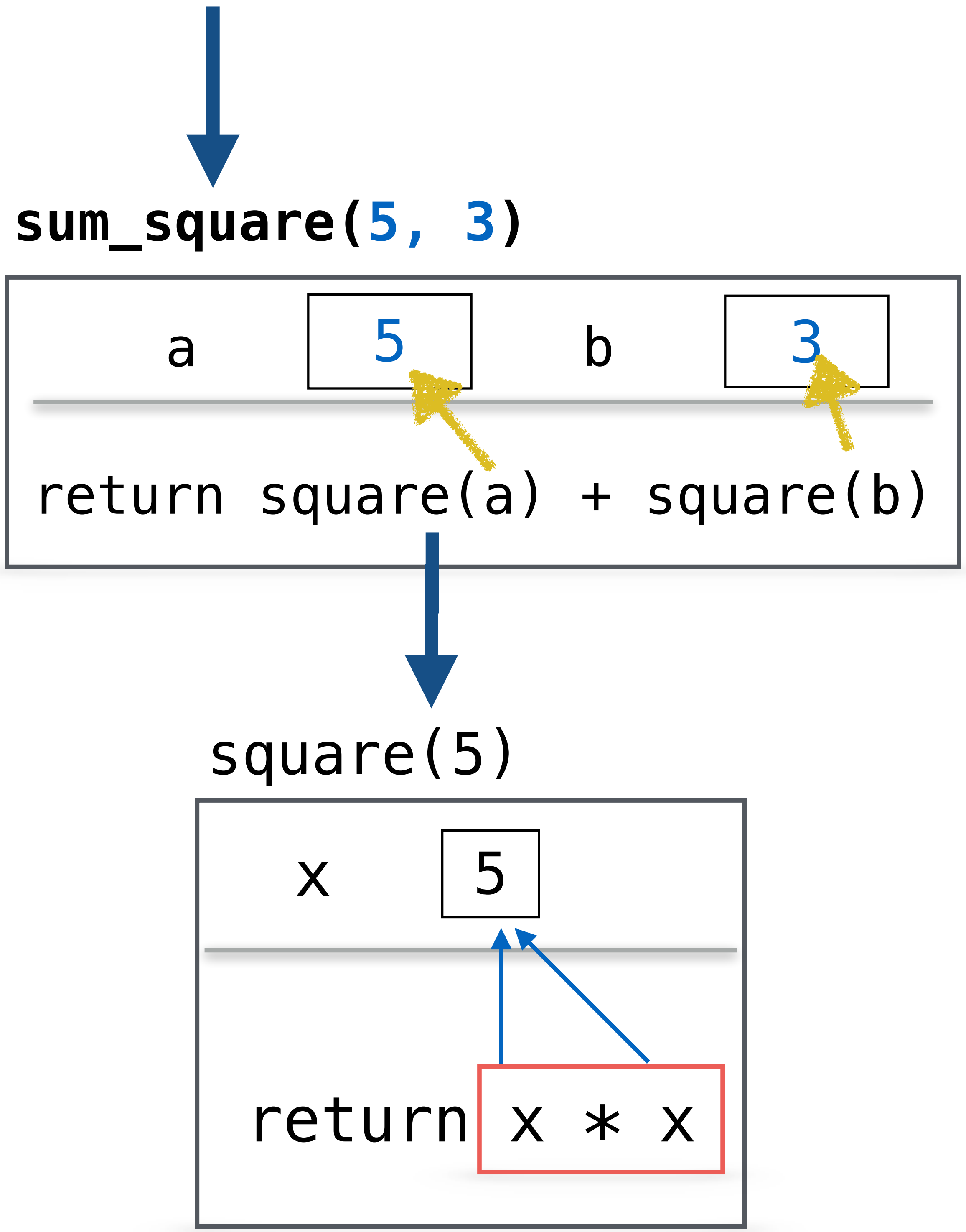
- How about functions that call other functions?

```
def sum_square(a, b):
    return square(a) + square(b)
```

- What happens when we call `sum_square(5, 3)`?

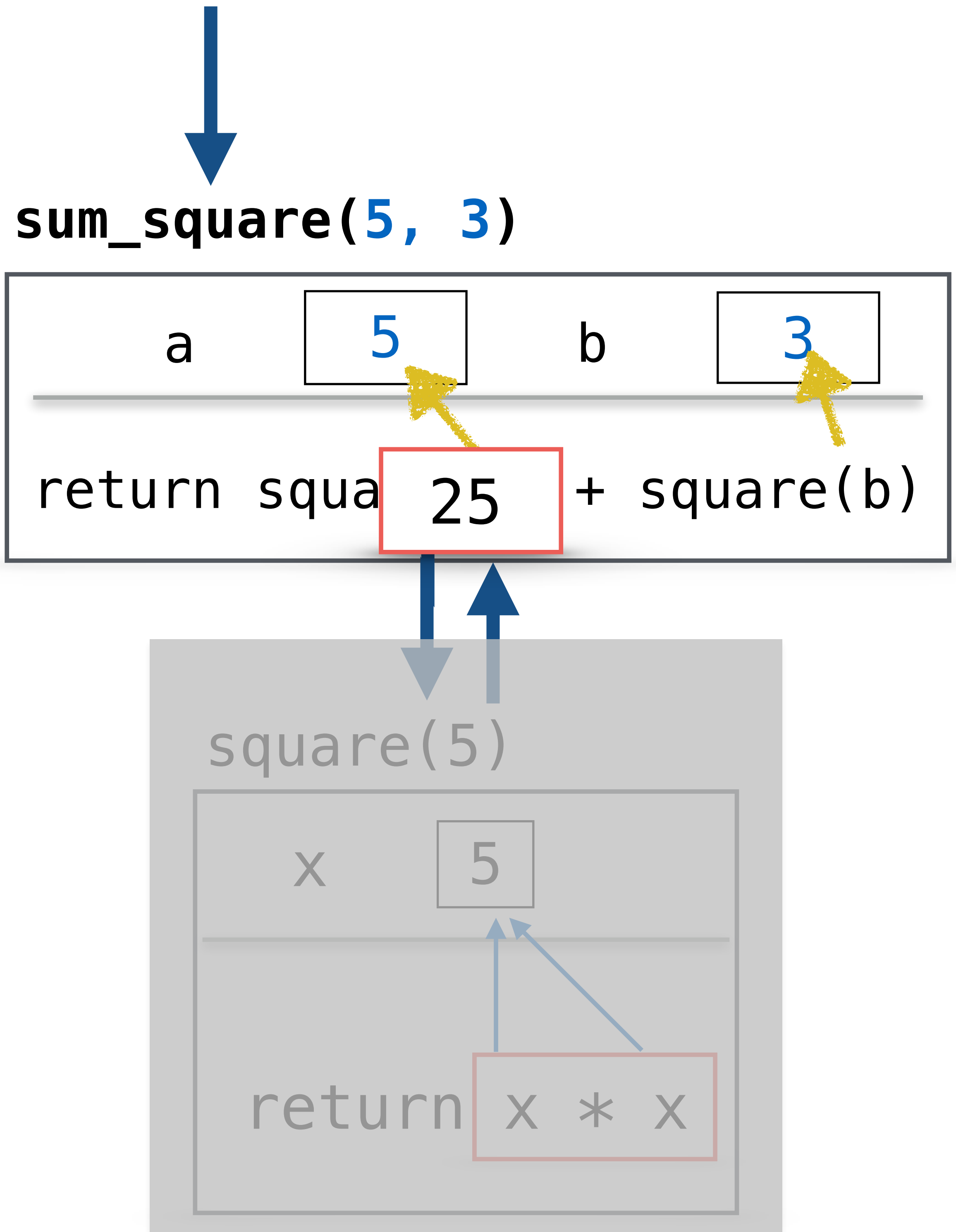
```
def sum_square(a, b):
   return square(a) + square(b)
```

```
>>> sum_square(5,3)
```



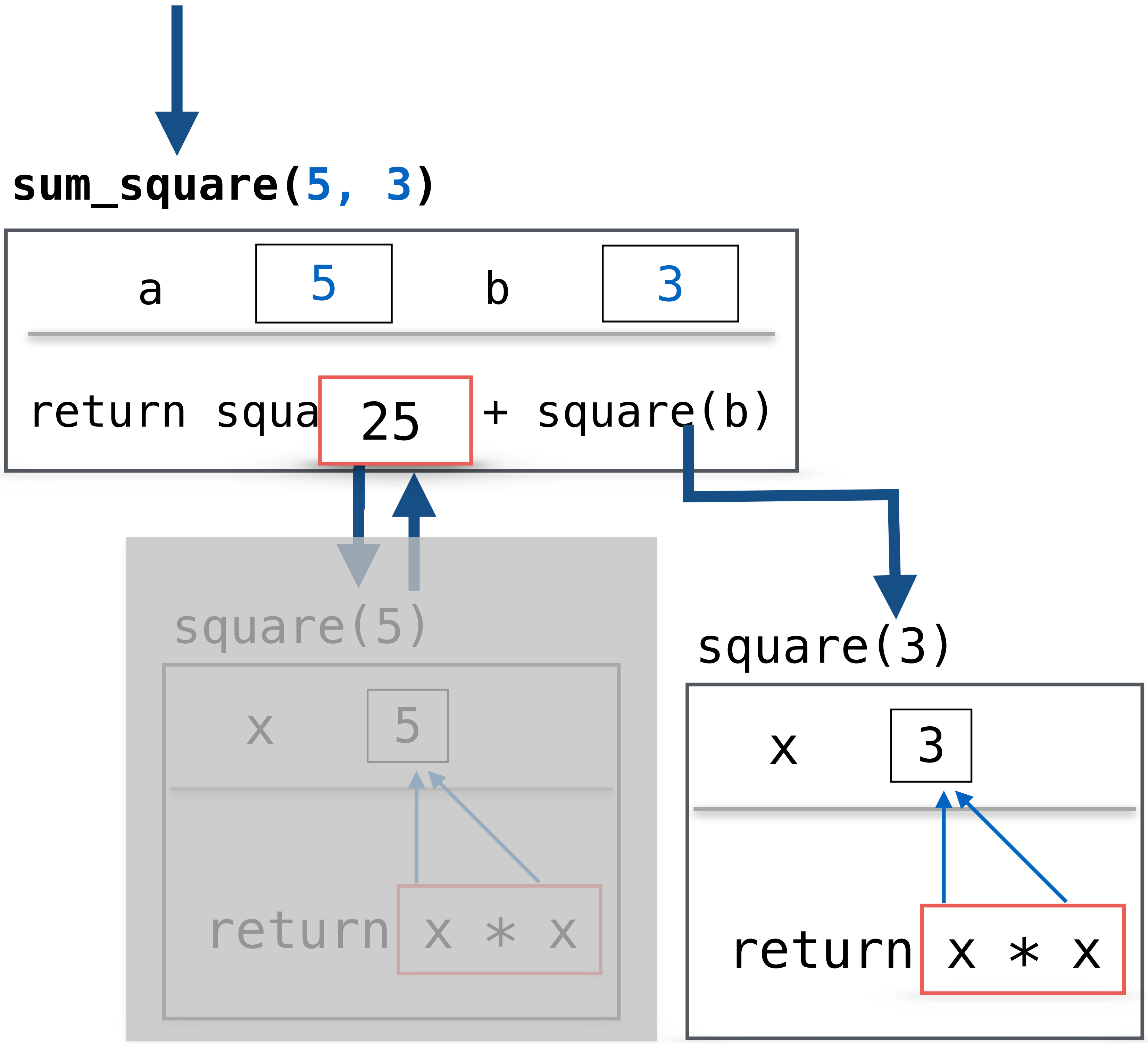
```
def sum_square(a, b):
   return square(a) + square(b)
```

```
>>> sum_square(5,3)
```



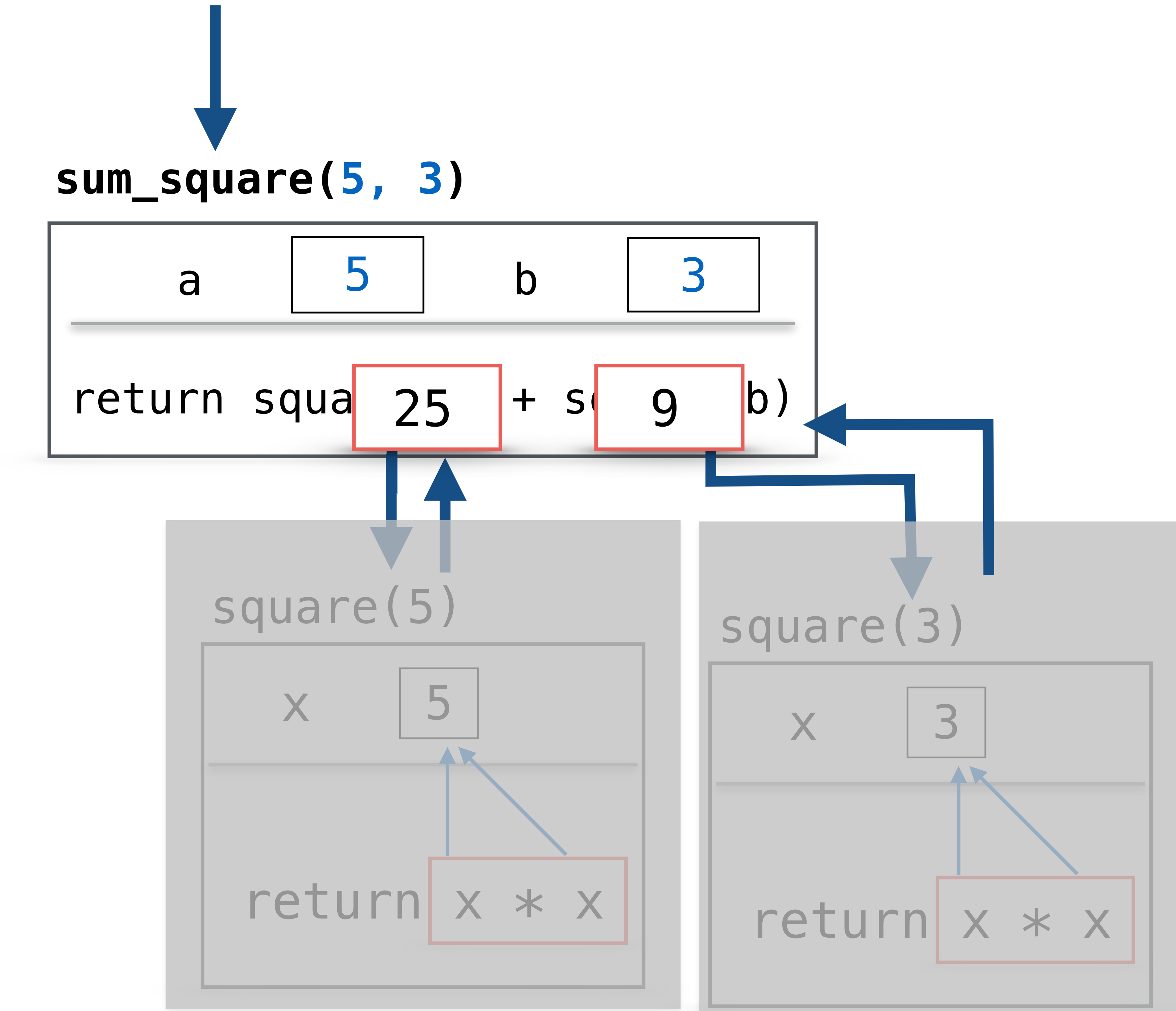
```
def sum_square(a, b):
   return square(a) + square(b)
```

```
>>> sum_square(5,3)
```

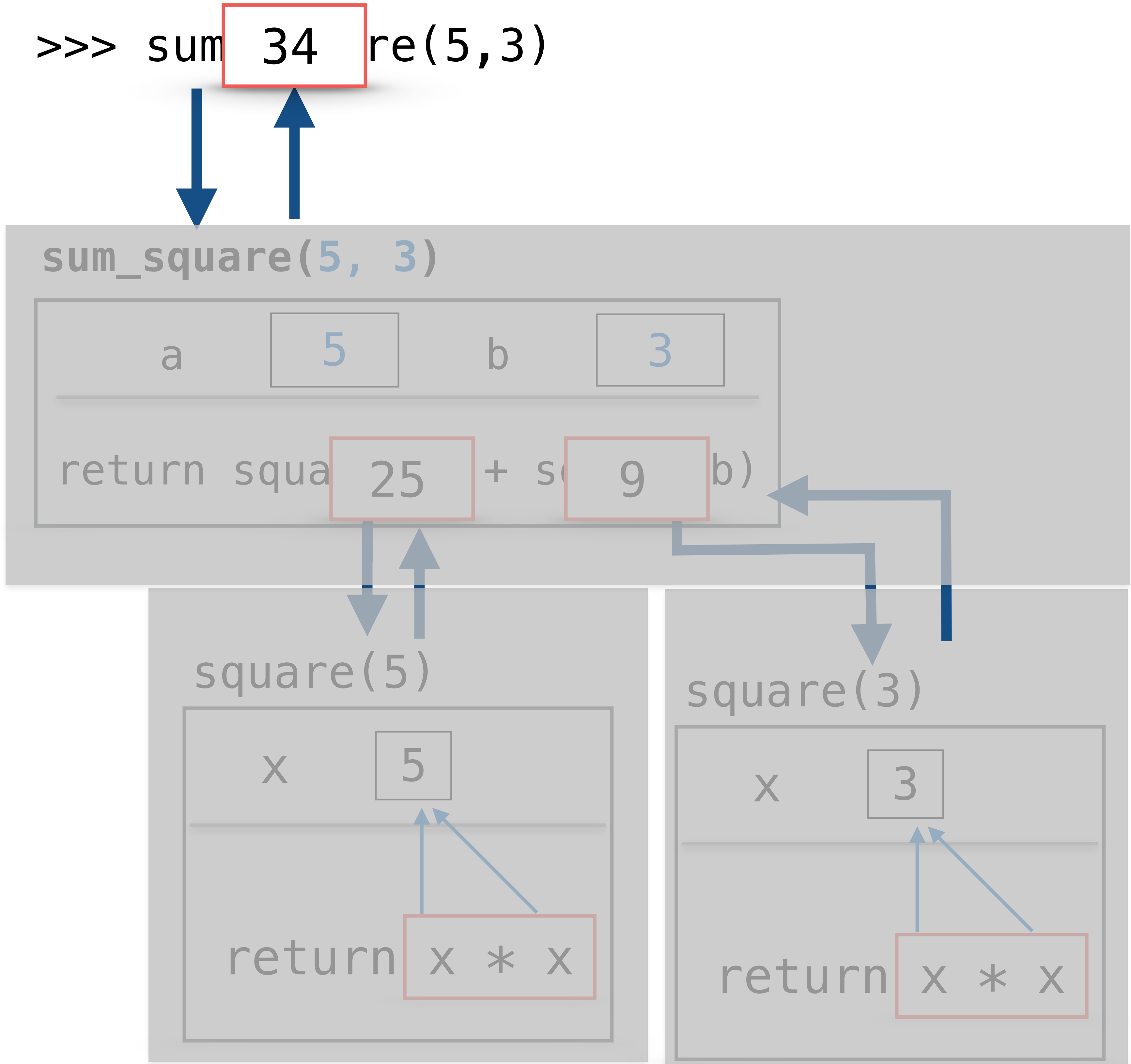


```
def sum_square(a, b):
   return square(a) + square(b)
```

```
>>> sum_square(5,3)
```



```
def sum_square(a, b):
   return square(a) + square(b)
```

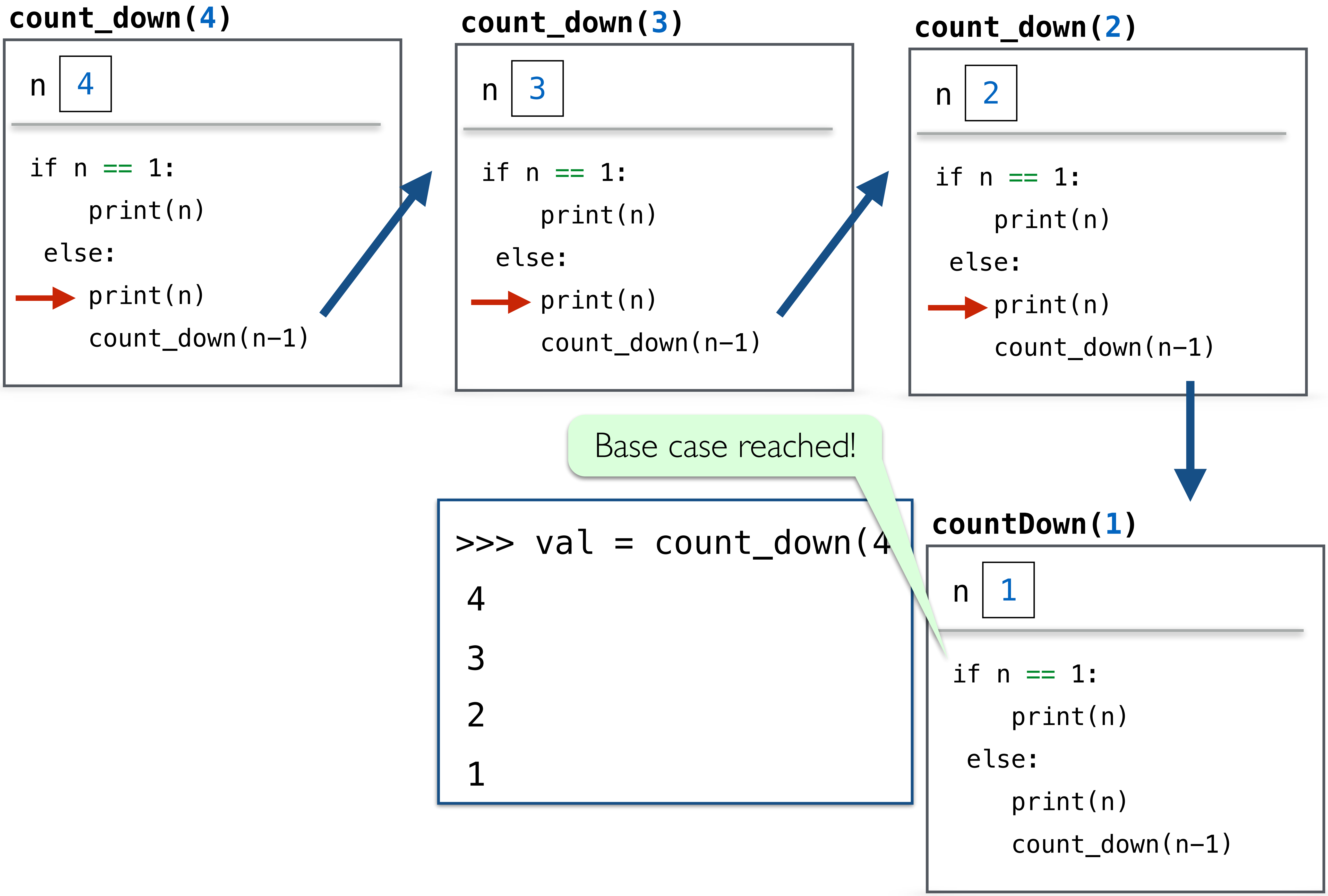


# Function Frame Model to Understand `count_down`

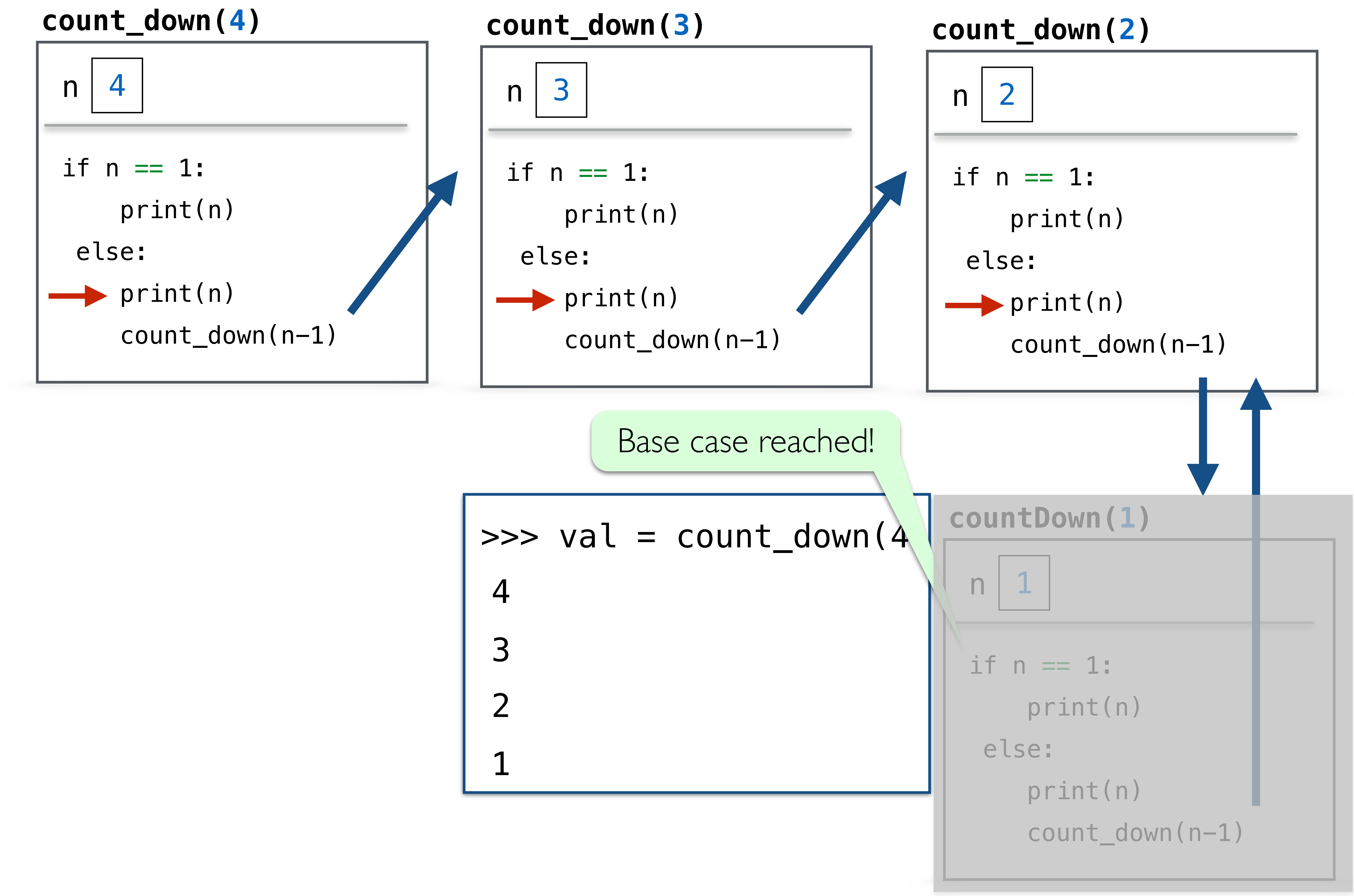
```python
def count_down(n):
    '''Prints ints from n down to 1'''
    if n == 1:
        print(n)
    else:
        print(n)
        count_down(n-1)
```

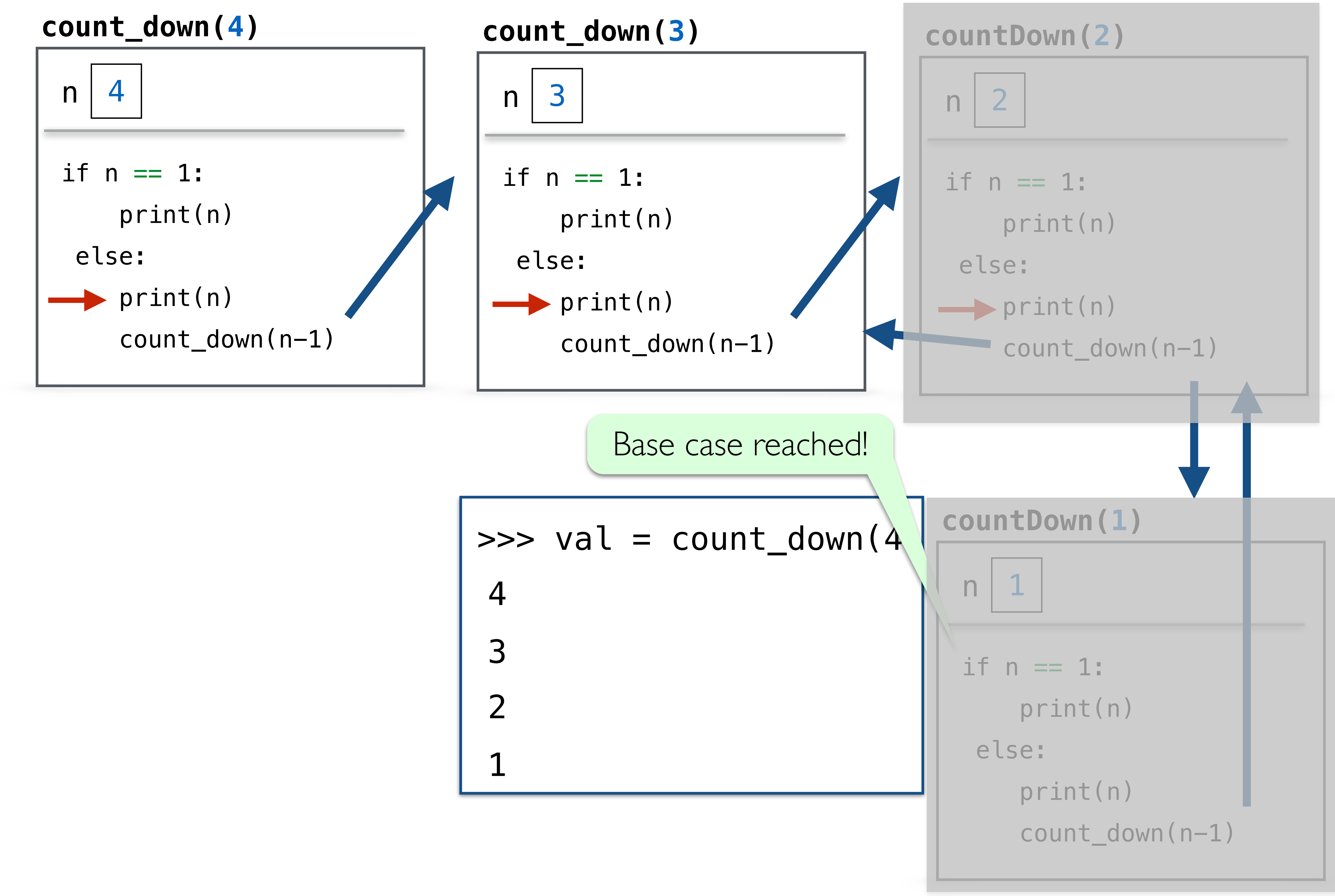
```
>>> val = count_down(5)
5
4
3
2
1
```

```
>>> val = count_down(4)
4
3
2
1
```

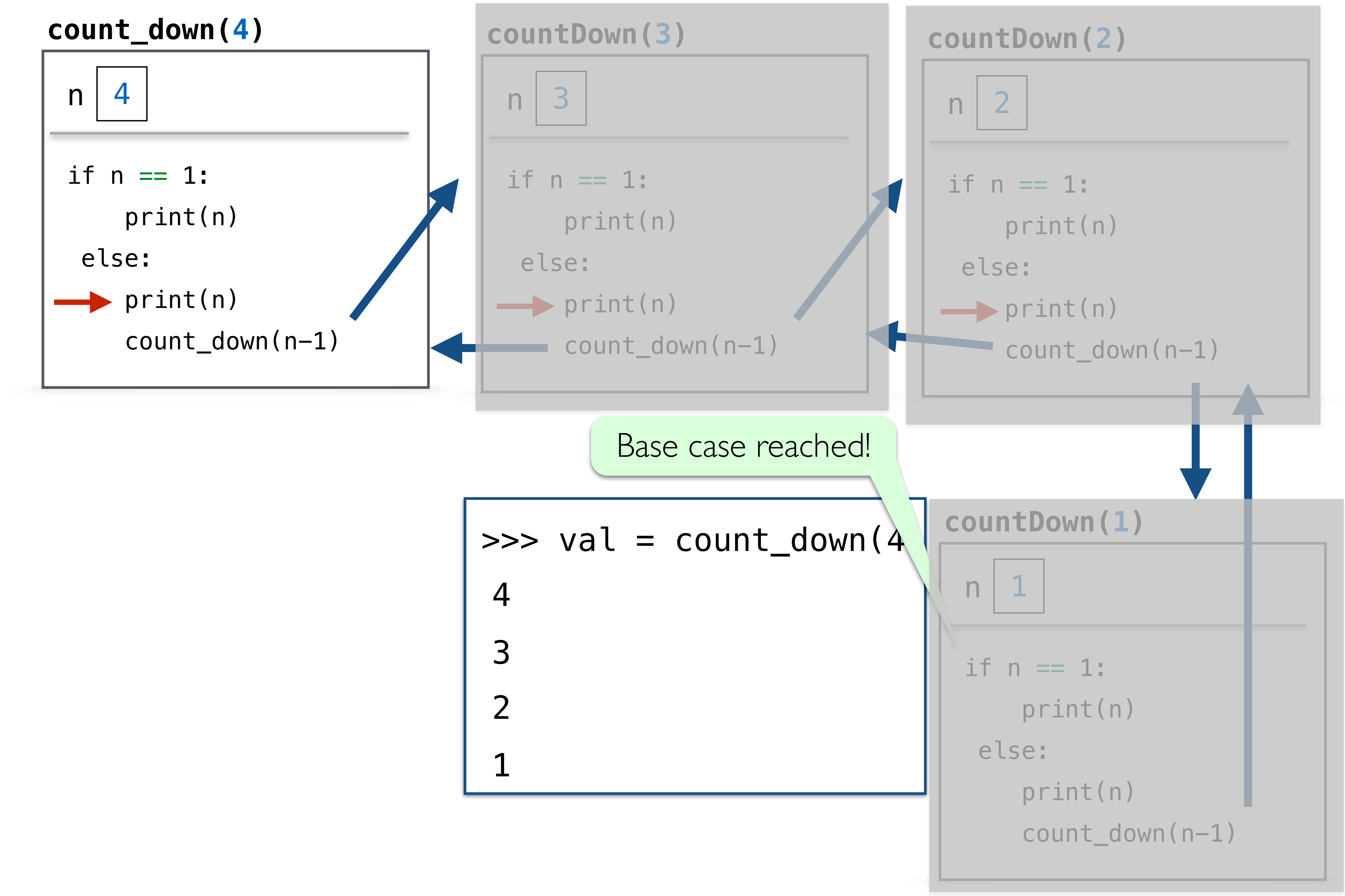


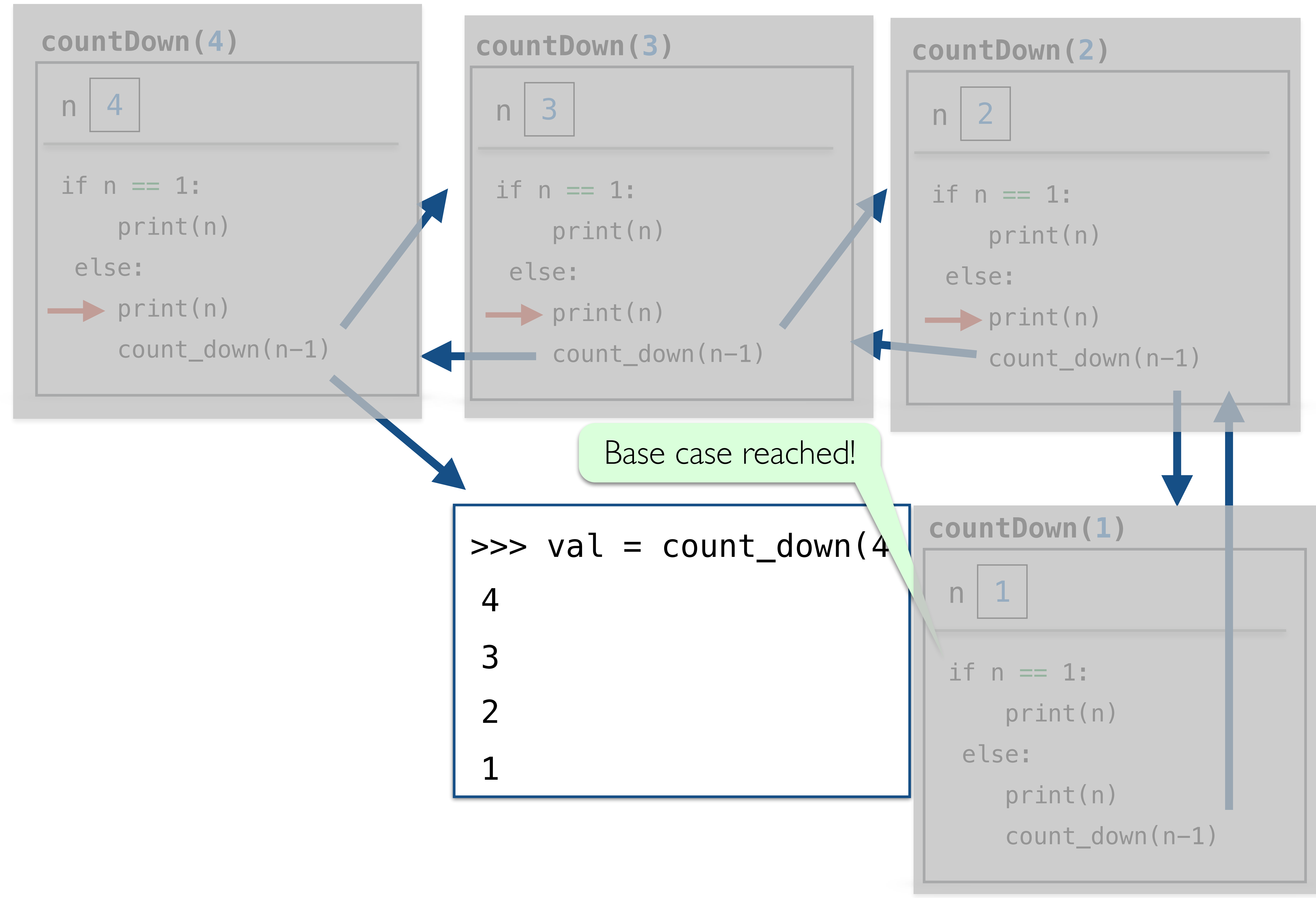
count_down(4)
n 4
if n == 1:
print(n)
else:
print(n)
count_down(n-1)
count_down(3)
n 3
if n == 1:
print(n)
else:
print(n)
count_down(n-1)
count_down(2)
n 2
if n == 1:
print(n)
else:
print(n)
count_down(n-1)
Base case reached!
>>> val = count_down(4
4
3
2
1
countDown(1)
n 1
if n == 1:
print(n)
else:
print(n)
count_down(n-1)


count_down(4)
n 4
if n == 1:
print(n)
else:
print(n)
count_down(n-1)
count_down(3)
n 3
if n == 1:
print(n)
else:
print(n)
count_down(n-1)
count_down(2)
n 2
if n == 1:
print(n)
else:
print(n)
count_down(n-1)
Base case reached!
>>> val = count_down(4
4
3
2
1
countDown(1)
n 1
if n == 1:
print(n)
else:
print(n)
count_down(n-1)



count_down(4)
n 4
if n == 1:
print(n)
else:
print(n)
count_down(n-1)
count_down(3)
n 3
if n == 1:
print(n)
else:
print(n)
count_down(n-1)
countDown(2)
n 2
if n == 1:
print(n)
else:
print(n)
count_down(n-1)
Base case reached!
>>> val = count_down(4
4
3
2
1
countDown(1)
n 1
if n == 1:
print(n)
else:
print(n)
count_down(n-1)


count_down(4)
n 4
if n == 1:
print(n)
else:
print(n)
count_down(n-1)
countDown(3)
n 3
if n == 1:
print(n)
else:
print(n)
count_down(n-1)
countDown(2)
n 2
if n == 1:
print(n)
else:
print(n)
count_down(n-1)
Base case reached!
>>> val = count_down(4
4
3
2
1
countDown(1)
n 1
if n == 1:
print(n)
else:
print(n)
count_down(n-1)


countDown(4)
n 4
if n == 1:
print(n)
else:
print(n)
count_down(n–1)
countDown(3)
n 3
if n == 1:
print(n)
else:
print(n)
count_down(n–1)
countDown(2)
n 2
if n == 1:
print(n)
else:
print(n)
count_down(n–1)
Base case reached!
>>> val = count_down(4
4
3
2
1
countDown(1)
n 1
if n == 1:
print(n)
else:
print(n)
count_down(n–1)

# Recursion GOTCHAs!

# GOTCHA #1

- If the problem that you are solving recursively **is not getting smaller**, that is, you are not getting closer to the base case --- **infinite recursion**!
- Never reaches the base case

```
def count_down_gotcha(n):
    '''Prints ints from 1 up to n’''
    if n == 1:  # Base case
        print(n)
    else:       # Recursive case
        print(n)
        count_down_gotcha(n)
```

Subproblem not getting smaller!

# GOTCHA #2

- Missing base case/unreachable base case--- another way to cause **infinite recursion**!

```
def print_halves_gotcha(n):
    """Prints n, n/2, down to ... 1"""
    if n >= 0:
        print(n)
        return print_halves_gotcha(n/2)
```

Always true!

# "Maximum recursion depth exceeded"

- In practice, the infinite recursion examples will terminate when Python runs out of resources for creating function call frames, leads to a "maximum recursion depth exceeded" error message

# Recursion vs. Iteration:
# `sum_list`

# `sum_list`

- **Goal:** Write a function to sum up a list of numbers
- Iterative approach? (i.e., using loops?)

# Iterative Approach to `sum_list`

- **Goal:** Write a function to sum up a list of numbers
- Iterative approach:

```
def sum_list_iterative(num_lst):
    sum = 0
    for num in num_lst:
        sum += num
    return sum
```

```
>>> sum_list_iterative([3, 4, 20, 12, 2, 20])
61
```

# sum_list

- **Goal:** Write a function to sum up a list of numbers
- Recursive approach?

# Recursive approach to `sum_list`

- **Base case:**
  - `num_lst` is empty, return `0`
- **Recursive rule:**
  - Return first element of `num_lst` plus result from calling `sum_list` on rest of the elements of the list.
- Example: Suppose `num_lst = [6, 3, 6, 5]`
  - `sum_list([6, 3, 6, 5]) = 6 + sum_list([3, 6, 5])`
  - `sum_list([3, 6, 5]) = 3 + sum_list([6, 5])`
  - `sum_list([6, 5]) = 6 + sum_list([5])`
  - `sum_list([5]) = 5 + sum_list([])`
- For the base case we have `sum_list([])` returns `0`

# Recursive approach to `sum_list`

- **Base case:**
  - `num_lst` is empty, return `0`
- **Recursive rule:**
  - Return first element of `num_lst` plus result from calling `sum_list` on rest of the elements of the list.
- Example: Suppose `num_lst = [6, 3, 6, 5]`
  - `sum_l` **20** `[6, 3, 6, 5]) = 6 + sum_l` **14** `[3, 6, 5])`
  - `sum_l` **14** `[3, 6, 5]) = 3 + sum_` **11** `t([6, 5])`
  - `sum_l` **11** `[6, 5]) = 6 + su` **5** `st([5])`
  - `sum_l` **5** `[5]) = 5 + s` **0** `ist([])`
- For the base case we have `sum_list([])` returns `0`

# Recursive approach to `sum_list`

```python
def sum_list(num_lst):
    """Returns sum of given list"""
    if num_lst == []:
        return 0
    else:
        return num_lst[0] + sum_list(num_lst[1:])
```

```
>>> sum_list([3, 4, 20, 12, 2, 20])
61
```

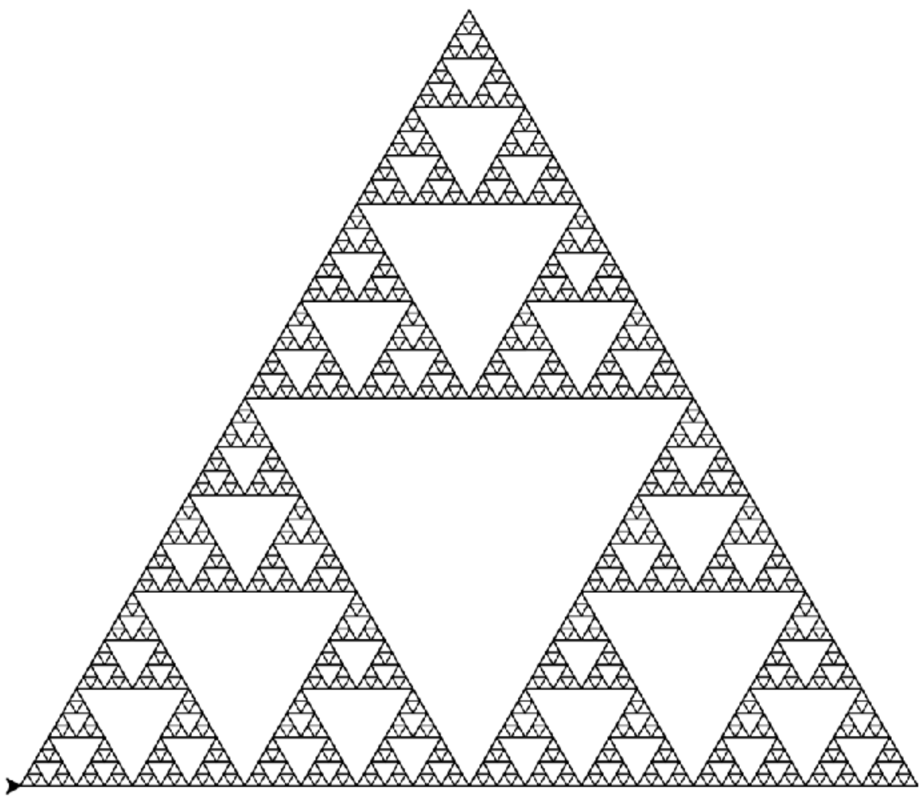
# Compare `sum_list` approaches

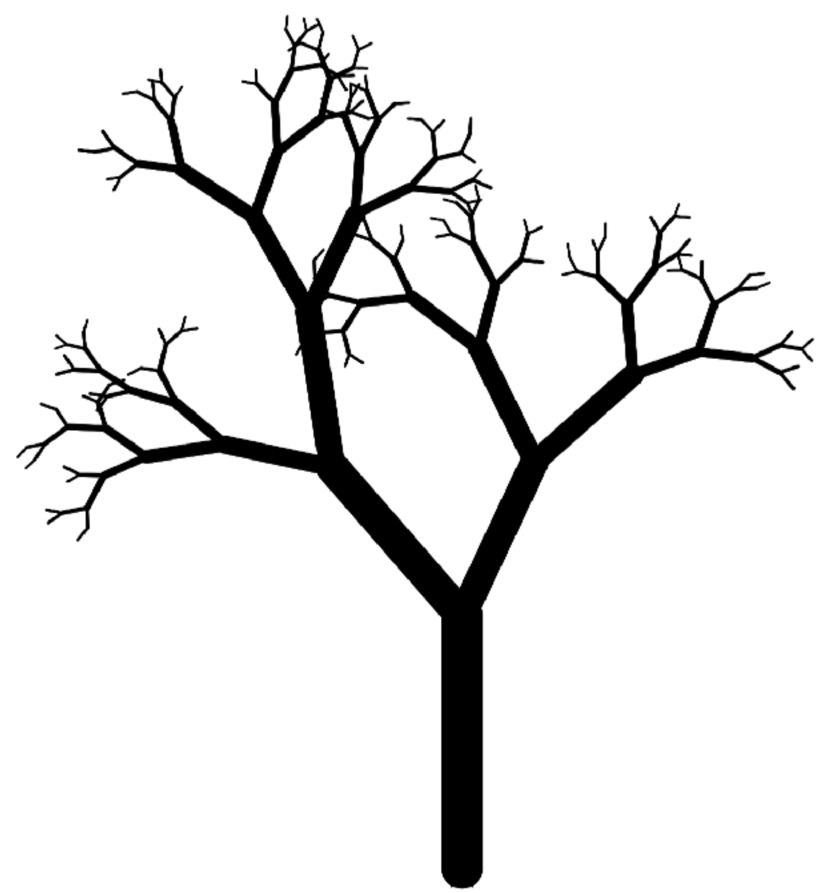
- Compare/Contrast:

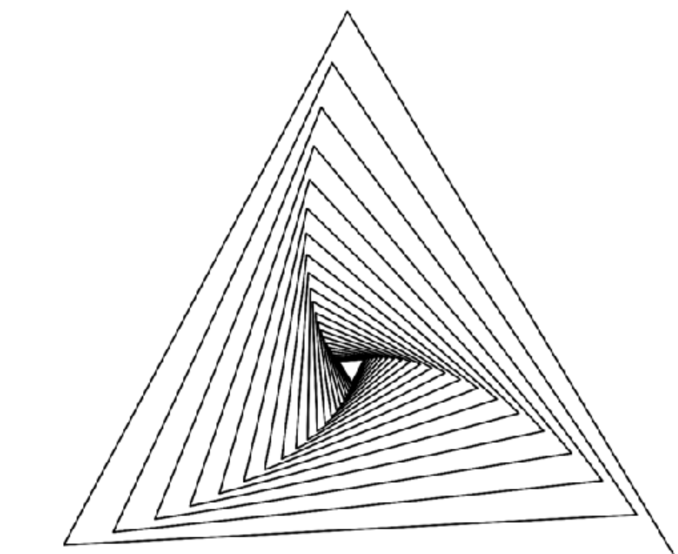
```
def sum_list_iterative(num_lst):
    sum = 0
    for num in num_lst:
        sum += num
    return sum
```

```
def sum_list(num_lst):
    if num_lst == []:
        return 0
    else:
        return num_lst[0] + sumList(num_lst[1:])
```

# Graphical Recursion







# The Turtle Module

- Turtle is a **graphics module** first introduced in the 1960s by computer scientists Seymour Papert, Wally Feurzig, and Cynthia Solomon.
- It uses a programmable cursor — fondly referred to as the "turtle" — to draw on a Cartesian plane (x and y axis.)

# Turtle In Python

- `turtle` is available as a built-in module in Python. See the Python turtle module API for details.
- Basic turtle commands:

Use `from turtle import *` to use these commands

| | |
|---|---|
| `fd(dist)` | turtle moves forward by dist |
| `bk(dist)` | turtle moves backward by dist |
| `lt(angle)` | turtle turns left angle degrees |
| `rt(angle)` | turtle turns right angle degrees |
| `up()` | (pen up) turtle raises pen in belly |
| `down()` | (pen down) turtle lowers pen from belly |
| `shape(shp)` | sets the turtle's shape to shp |
| `speed(spd)` | sets the turtle's speed 1-10 (slow-fast). 0 skips animation. |
| `home()` | turtle returns to (0,0) (center of screen) |
| `clear()` | delete turtle drawings; no change to turtle's state |
| `reset()` | delete turtle drawings; reset turtle's state |
| `setup(width, height)` | create a turtle window of given width and height |

# Basic Turtle Movement

- `forward(dist)` or `fd(dist)`, `left(angle)` or `lt(angle)`, `right(angle)` or `rt(angle)`, `backward(dist)` or `bk(dist)`

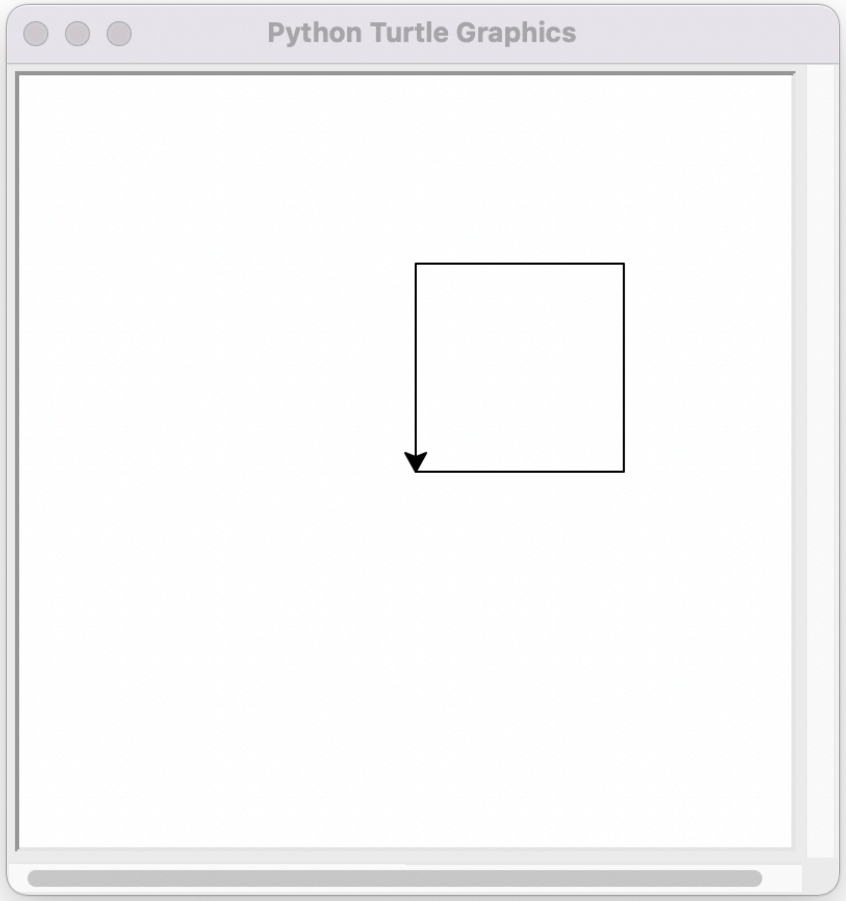
```
# set up a 400x400 turtle window
setup(400, 400)
reset()

fd(100) # move the turtle forward 100 pixels

lt(90) # turn the turtle 90 degrees to the left

fd(100) # move forward another 100 pixels

# complete a square
lt(90)
fd(100)
lt(90)
fd(100)
done()
```

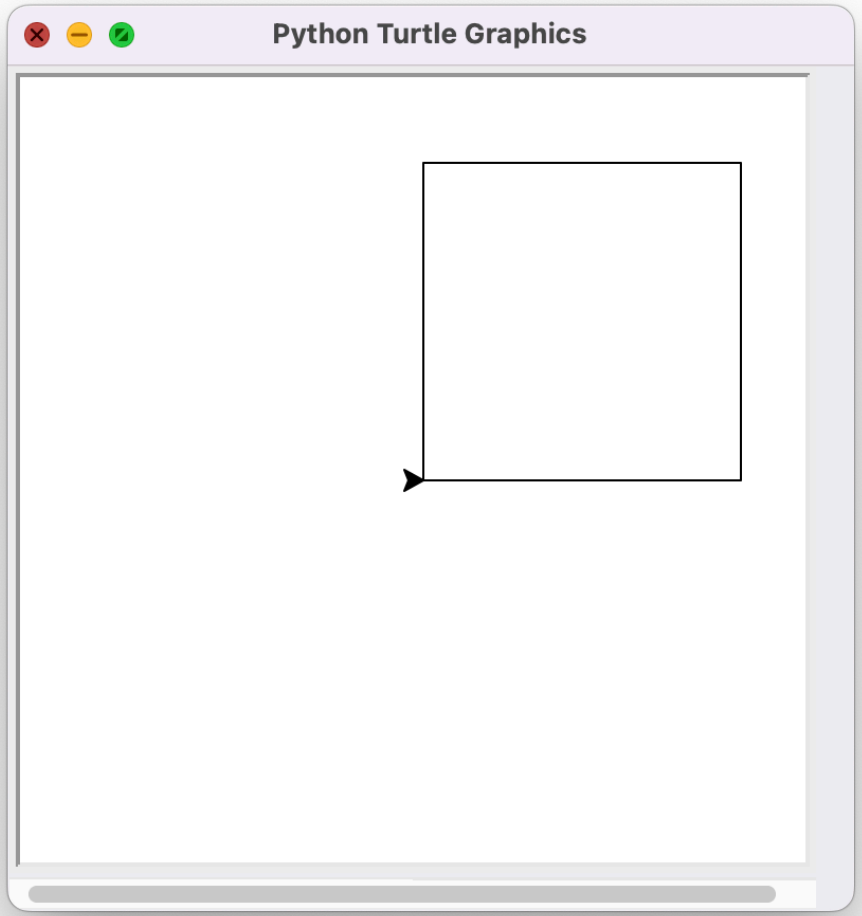


# Drawing Basic Shapes With Turtle

- We can write functions that use turtle commands to draw shapes.
- For example, here's a function that draws a square of the desired size

```
def draw_square(length):
    # a loop that runs 4 times
    # and draws each side of the square
    for i in range(4):
        fd(length)
        lt(90)
    done()
```

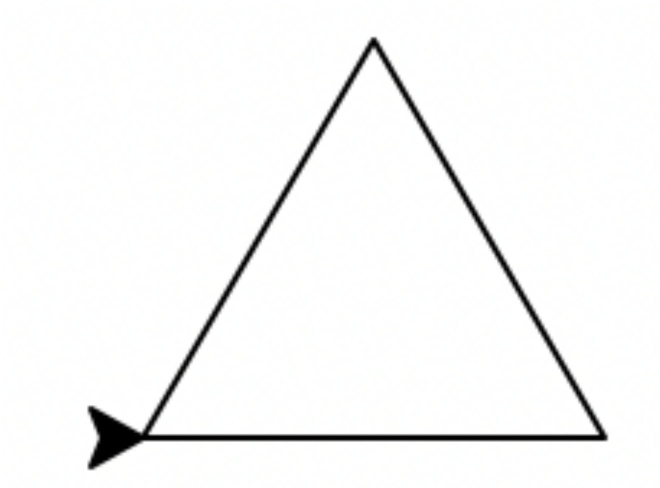
```
setup(400, 400)
reset()
draw_square(150)
```



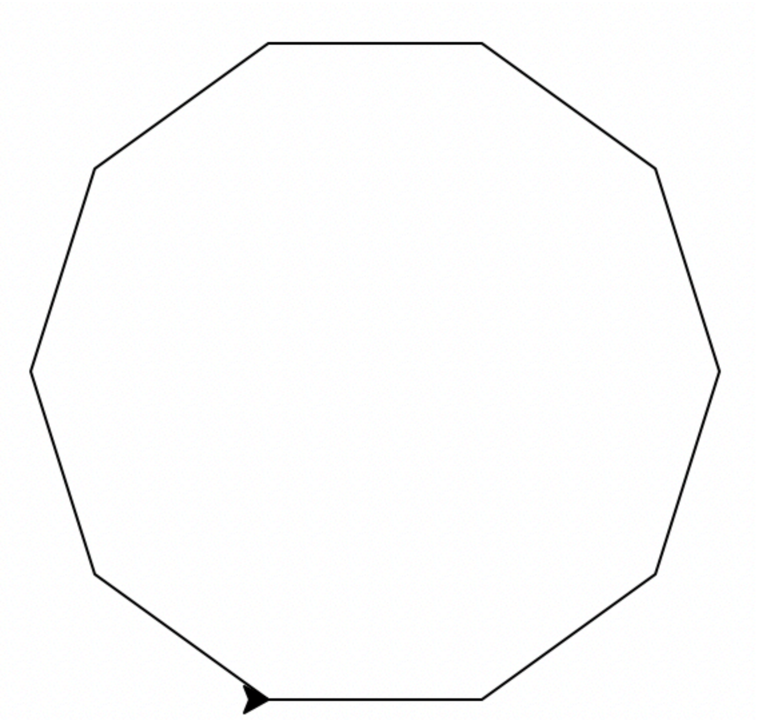
# Drawing Basic Shapes With Turtle

- How about drawing polygons?

```
def draw_polygon(length, num_sides):
    for i in range(num_sides):
        fd(length)
        lt(360/num_sides)
    done()
```



`draw_polygon(80, 3)`



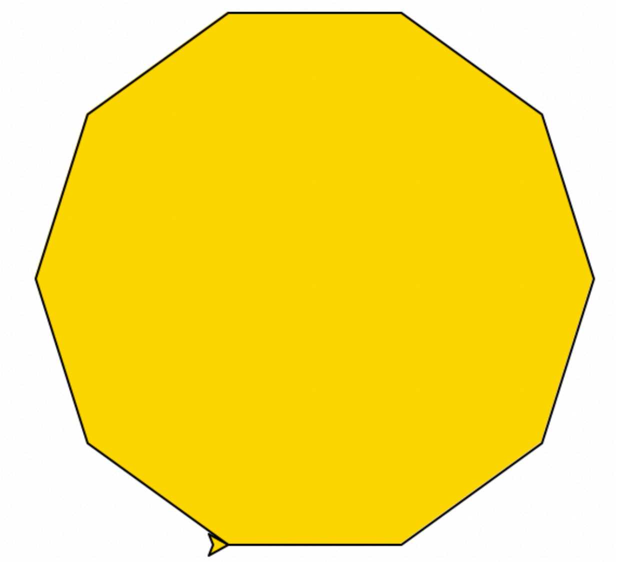
`draw_polygon(80, 10)`

# Adding Color!

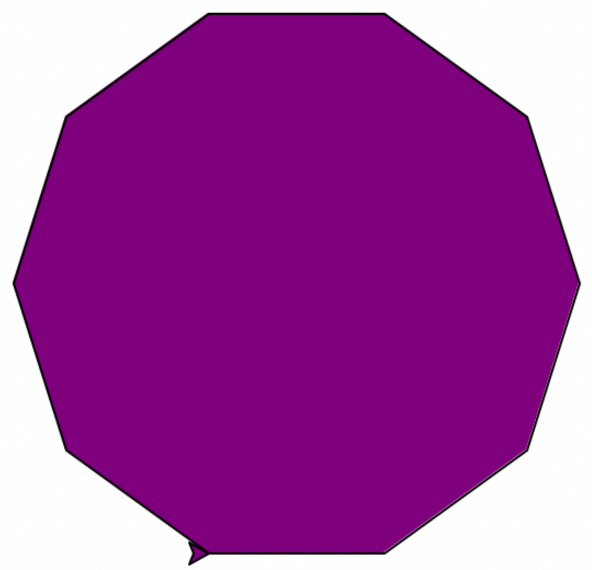
- What if we wanted to add some color to our shapes?

```
def draw_polygon_color(length, num_sides, color):
    # set the color we want to fill the shape with
    # color is a string
    fillcolor(color)

    begin_fill()
    for i in range(num_sides):
        fd(length)
        lt(360/num_sides)
    end_fill()
    done()
```



```
draw_polygon_color(80, 10, "gold")
```



```
draw_polygon_color(80, 10, "purple")
```

# Next Time: Recursive Figures With Turtle

- Next time we will explore how to draw recursive pictures with Turtle