First-Order Logic as a Knowledge Representation

A more expressive formalism than Propositional Logic.

Allows us to describe:
- objects: things with identities
- properties: descriptions of objects
- relations: connections among objects
- functions: relations for which there is only one "value" for a given "input"

Objects. People, kitchen utensils, colleges, cars, etc.
Properties. Red, Sharp, Mean-spirited, Loyal, etc.
Relations. HasColor, Hates, Contains, SiblingOf, etc.
Functions. FatherOf, PrimaryResidenceOf, etc.

Syntax of First-Order Logic

Def. A term, which represents an object, is either a constant, a variable, or the result of a function applied to a term.

A predicate symbol – i.e., either a property or a relation – followed by parenthesized terms is an atomic sentence. It describes a fact.

    Ex. BrotherOf(Joe, Harry) describes the fact: Joe is a brother of Harry.

The negation of a sentence is also a sentence.

Sentences combined with logical connectives ($\wedge$ $\vee$ $\Rightarrow$ $\Leftrightarrow$) are sentences.

Quantified sentences are sentences.

    Ex. $\forall$x Attends(x, Williams) $\Rightarrow$ Smart(x)
        $\exists$x Attends(x, Williams) $\wedge$ Plays(x, UltimateFrisbee)

Semantics

Terms, which describe objects, have the meaning that is assigned to them by the creator of the term:

    Ex. Bob refers to "Bob"; Car(Bob) refer's to Bob's car, assuming that a person has at most one car.

Predicates, which describe properties and relations, have the meaning assigned to them by the creator of the predicate.

An atomic sentence is true if the relation or property referred to by the predicate symbol is true of the terms to which it is applied.

The meaning of a complex sentence (i.e., whether it is true or false) is defined by the rules for combining complex sentences. (Recall the truth table for propositional logic).

Examples.

Marcus was a man.
    Man(Marcus)

Marcus was a Pompeiian.
    Pompeiian(Marcus)

Caesar was a ruler.
    Ruler(Caesar)

Marcus tried to assassinate Caesar.
    TryAssassinate(Marcus, Caesar)

Marcus was not loyal to Caesar.
    ¬LoyalTo(Marcus, Caesar)

The Meaning of Quantified Sentences

Quantifiers allow us to express properties of entire collections of objects.

∀x Pompeiian(x) ⇒ Roman(x)
    has the meaning:

    Pompeiian(Caesar) ⇒ Roman(Caesar) ∧
    Pompeiian(Marcus) ⇒ Roman(Marcus) ∧
    Pompeiian(Vase1) ⇒ Roman(Vase1) ∧
    …

∃y LoyalTo(Joe, y)
    has the meaning:

    LoyalTo(Joe, Marcus) ∨
    LoyalTo(Joe, Caesar) ∨
    …

More examples.

All Romans were either loyal to Caesar or hated Caesar.

$\forall x\ Roman(x) \Rightarrow LoyalTo(x, Caesar) \vee Hate(x, Caesar)$

Everyone is loyal to someone.

$\forall x\ \exists y\ LoyalTo(x, y)$

People try to assassinate rulers they are not loyal to.

$\forall x\ \forall y\ Person(x) \wedge Ruler(y) \wedge TryAssassinate(x, y) \Rightarrow \neg LoyalTo(x,y)$

Equality

Equality allows us to refer to a single object by different names.

Ex. WorstEnemy(Caesar) = Marcus.

Note that this is also a valid atomic sentence with the meaning

Caesar's worst enemy was Marcus.

Equality can also be used with negation.

Ex. $\neg$(WorstEnemy(Caesar) = Cleopatra).

This can also be expressed as

WorstEnemy(Caesar) $\neq$ Cleopatra

Exercise. Consider this set of sentences. How would you express them in First-Order Logic?

The members of the Elm Street Bridge Club are Joe, Sally, Bill, and Ellen.
Joe is married to Sally.
Bill is Sally's brother.
The spouse of every married person in the club is also in the club.
Unmarried members of the club do not host meetings in their homes.
The last meeting of the club was at Bill's house.

Is this enough to conclude that Bill and Ellen are married?
