

## Lecture 33

Finishing up from last time

Thm. The following problems about Turing machine are undecidable:

- 1) Given a Turing machine  $M$  and an input string  $w$ , does  $M$  halt on input  $w$ ?
- 2) For a certain fixed machine, given an input string  $w$ , does  $M$  halt on input  $w$ ?
- 3) Given a Turing machine  $M$ , does  $M$  halt on the empty tape?
- 4) Given a Turing machine  $M$ , is there any string at all on which  $M$  halts?
- 5) Given a Turing machine  $M$ , does  $M$  halt on every input string?
- 6) Given 2 machines  $M_1$  and  $M_2$ , do they halt on the same input strings?
- 7) Given a Turing machine  $M$ , is the language  $M$  accepts regular? context free? decidable?

So what can you do?

From the homework

### 5.4.1

(a) Let  $M = (K, \Sigma, \delta, s, H)$  be a Turing machine.

Let  $q = |K|$  i.e., the total number of possible states.

Let  $c = |\Sigma|$  i.e., the total number of tape symbols, including the blank symbol and  $>$

Now consider the total number of configurations that are possible using  $k$  tape squares:

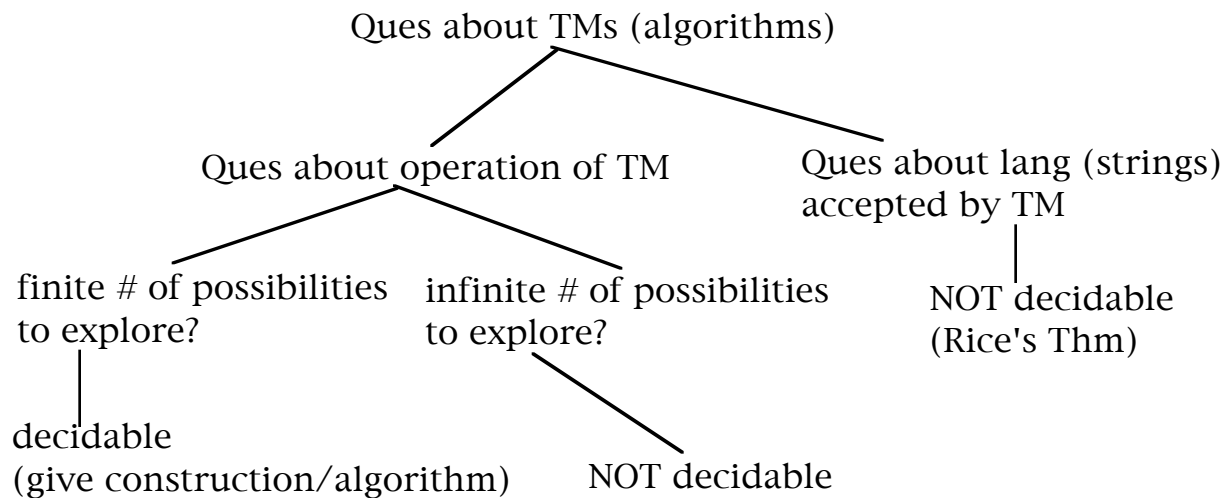
$t \leq (q)(c^k)(k)$ , where the final  $k$  in the multiplication refers to the possible head positions. ( $\leq$  due to the fact that  $>$  should never appear in more than the leftmost tape square.)

To check whether the Turing machine  $M$  uses  $k$  tape squares on a given input  $w$ :

- simulate M on w for at most t steps.
- if M has used k tape squares in that time, answer Y;
- if not, answer N [M must be in a loop and will continue to cycle without reaching k tape squares]

How can you decide whether a problem is decidable or not? Here are some heuristics:

**decidable vs undecidable problems:**



Rice's Theorem. Any non-trivial property of recursively enumerable languages is undecidable.

- ex.
- emptiness
  - finiteness
  - regularity
  - context free-dom

Def. a trivial property is one that is either true for all r.e. languages or false for all r.e. languages.