Lecture 32

Homework #32: 5.4.1, 5.4.2 – turn in 5.4.1 b, 5.4.2 a and d

We have a number of different ways to show that a language is decidable. We can:

- Directly present a Turing Machine that decides the language. Here we can select from any of the "complex" models that use multiple tapes, etc. We can also use simple Turing Machines to construct more complicated machines.
- Make use of the closure theorem from the previous lecture.

Today we will explore a mechanism by which we can prove that a language is not decidable. (and no, it's not another Pumping Theorem...)

The primary technique we will explore is **reduction**.
To show that L_1 is undecidable:

Identify L_2 that is undecidable.
Reduce L_2 to L_1. Show that if L_1 were decidable,
L_2 would be as well. (Finding an algorithm for L_2 **reduces to** finding an algorithm for L_1.)

Thm. The following problems about Turing Machines (i.e., algorithms) are undecidable:

1) Given a Turing machine M and an input string w, does M halt on input w?
2) For a certain fixed machine, given an input string w, does M halt on input w?
3) Given a Turing machine M, does M halt on the empty tape?
4) Given a Turing machine M, is there any string at all on which M halts?
5) Given a Turing machine M, does M halt on every input string?
6) Given 2 machines M_1 and M_2, do they halt on the same input strings?
7) Given a Turing machine M, is the language M accepts regular? context free? decidable?

Proof.

1) This is the Halting Problem. Recall proof from 1st class (which was essentially a diagonalization proof).

2) Let L_h = {<M,w> : M accepts - i.e., halts on - w}

Now consider M^1, a machine that semidecides L_h. That is, it halts when given <M,w> such that M halts on w. It does not halt otherwise.

If we choose M^1 to be the fixed machine, then we can decide L_h.

3) Given M, does M halt on the empty tape?

Let L_h = {<M,w> : M accepts - i.e., halts on - w}

Suppose we could find M_0 to decide
{<M> : M accepts e}

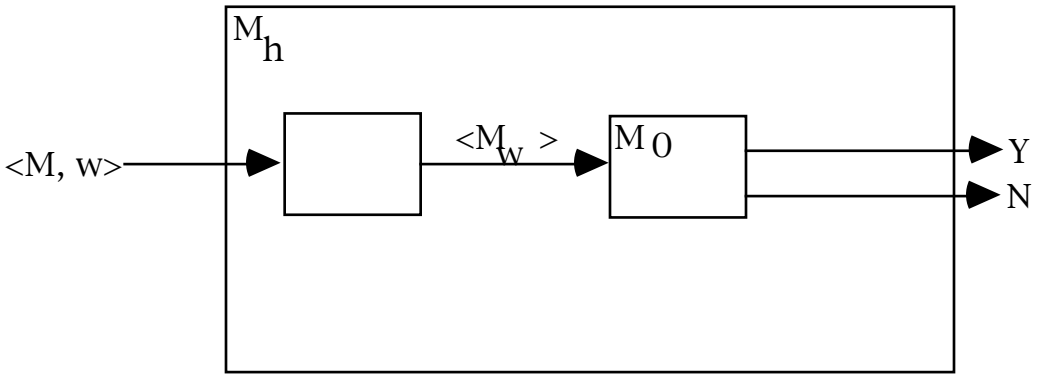
We will now show that M_0 could be used to decide L_h.

Claim: Given a Turing machine M and input w, there is a systematic way to construct M_w that operates as follows: M_w, when started with empty tape (s, ##), writes w and then simulates M on w.

Now we can use M_0 to decide L_h.

Given M and w, construct M_w.

Deliver <M_w> as input to M_0.
if M_0 halts with **Y** on the tape, then M halts on w
if M_0 halts with **N** on the tape, then M does not halt on w.



4) Given M, is there any string at all on which M halts?

Let L_e = {<M> : M halts on e}

Suppose we could construct M_{any} to decide
{<M> : M halts on some string}

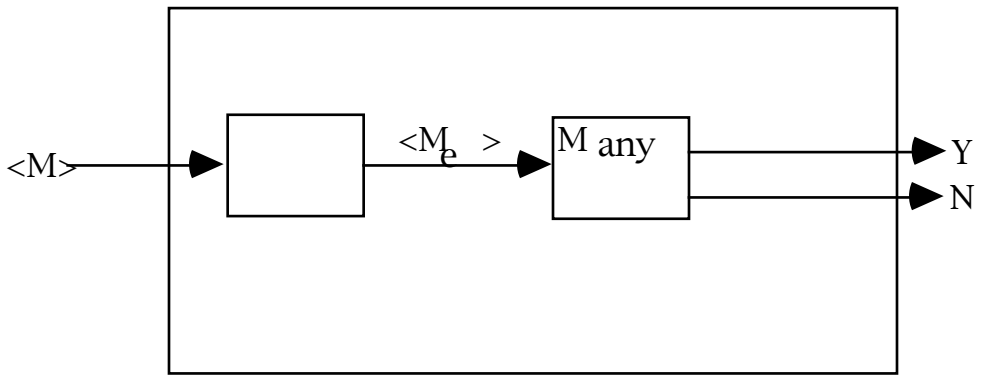
We will now show that M_{any} could be used to decide L_e.

Claim: Given a Turing machine M, there is a systematic way to construct M_e that operates as follows: M_e, when started with any input (s, #w#), erases w and then simulates M on e.

Now we can use M_{any} to decide L_e.

Given M, construct M_e.

Deliver <M_e> as input to M_{any}.
if M_{any} halts with **Y** on the tape, then M halts on something, which means that M halts on e.
if M_{any} halts with **N** on the tape, then M does not halt on anything, which means that M does not halt on e.



5) Same as 4.

6) Given M_1 and M_2, do they halt on the same input strings?

Let L_{all} = {<M> : M halts on all input}

Suppose we could find M_{same} to decide
{<M_1, M_2> : M_1 and M_2 halt on the same input strings}

We will now show that M_{same} could be used to decide L_{all}.

First create a machine M_i that halts immediately on all input.

Now we can use M_{same} to decide L_{all}.

Given M, input <M, M_i> to M_{same}.

But we know that L_{all} is undecidable. Therefore, this problem is undecidabe as well.

7) See text.