Lecture 2

Homework #2: 1.2.3, 1.3.1 (for R • R), 1.3.2a "S" only, 1.3.4, 1.3.5, 1.3.7, 1.3.9

TRUE & FALSE - important to review what it means for a statement to be true or false - especially since we're going to be concerned with proving statements to be one or the other

Intuitive notion of TRUE:

This class meets in TCL 206

What happens when you start combining sentences:

$p \wedge q$	p	q
T	T	T
F	T	F
F	F	T
F	F	F

$p \vee q$	p	q
T	T	T
T	T	F
T	F	T
F	F	F

if p then q $(\neg p \vee q)$	p	q
T	T	T
F	T	F
T	F	T
T	F	F

Proving (if p then q)

1) assume that p is true; show that q is also true
2) prove by contradiction: assume q to be false; show that you get an inconsistency with p.

Note that (q if p) means the same thing

But (q only if p) does not. (q only if p) means (if q then p)
(if q then p) is the **converse** of (if p then q)

So... (p **iff** q) = (if p then q) $\wedge$ (if q then p)

p iff q	p	q
T	T	T
F	T	F
F	F	T
T	F	F

Need to prove each of the two parts

Now, some notes on **sets**

2 sets are equal iff they have the same elements.
To prove 2 sets equal: Show that
    $A \subseteq B$ **and**
    $B \subseteq A$

There are various ways in which sets can be combined: you know them, and they're in the book (union, intersection, ...)

Laws about the ways in which sets combine:

Example. Distributive Laws

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

To prove this, will show that:
1) $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ **and**
2) $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$

1) Let $x \in A \cup (B \cap C)$
   Then x is in A, or x is in B and C.

   if x is in A, then $x \in (A \cup B)$;
   and $x \in (A \cup C)$.
   But then $x \in (A \cup B) \cap (A \cup C)$.

   if x is in B and C, then
   $x \in (A \cup B)$;
   and $x \in (A \cup C)$.
   But then $x \in (A \cup B) \cap (A \cup C)$.

   So $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

2) Let $x \in (A \cup B) \cap (A \cup C)$
Then x is in $(A \cup B)$ and x is in $(A \cup C)$
So x is in A or x is in B and C.
i.e., $x \in A \cup (B \cap C)$

So $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$

From 1) and 2) it follows that
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \times B$ is the Cartesian Product of 2 sets:
all ordered pairs (a,b) s.t. $a \in A$, $b \in B$.

A (binary) **relation** is a subset of $A \times B$
(in basic terms, it's a set of items that are "related to" each other in a particular way)

A **function** from A to B is a relation where for each $a \in A$ there is one and only one ordered pair with first component a.
[f(a) has only one value]

{*Explain how the difference between relations and functions will be illustrated by the models of computation we'll be considering*}

f: $A \to B$ is **onto** B if each element in B is f(a) for some a.
f: $A \to B$ is **one-to-one** if for all $a \neq a_1$, $f(a) \neq f(a_1)$.
f: $A \to B$ is a **bijection** if it is one-to-one and onto.

Probably more familiar with functions than with relations:

Relations can be combined in a manner similar to functions:

Example. Let R = {(a,b), (a, c), (c, d), (a, a), (b, a)}

$R \circ R$ = {(a, a), (a, d), (a, b), (a, c), (b, b), (b, c), (b, a)}
composition

R^{-1} = {(b, a), (c, a), (d, c), (a, a), (a, b)}

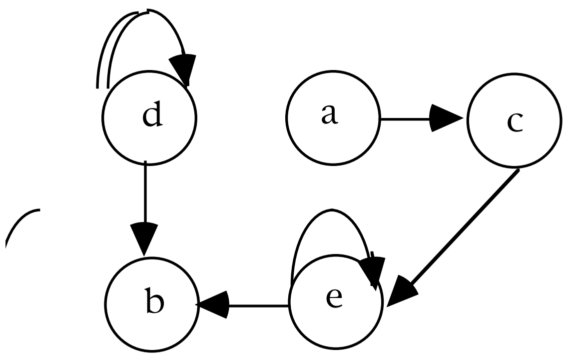
Now, let's consider a special type of binary relation:

$R \subseteq A \times A$ (a relation on a set and itself) (can also say A^2)

Can represent R by a directed graph. This might be useful for visualizing properties of R.
(Pictorial representations of various automata will follow this model.)

each element of A is a node;
arc from $a \rightarrow b$ iff $(a,b) \in R$.

Example. $\{(a,c), (c,e), (e,e), (e,b), (d,b), (d,d)\}$



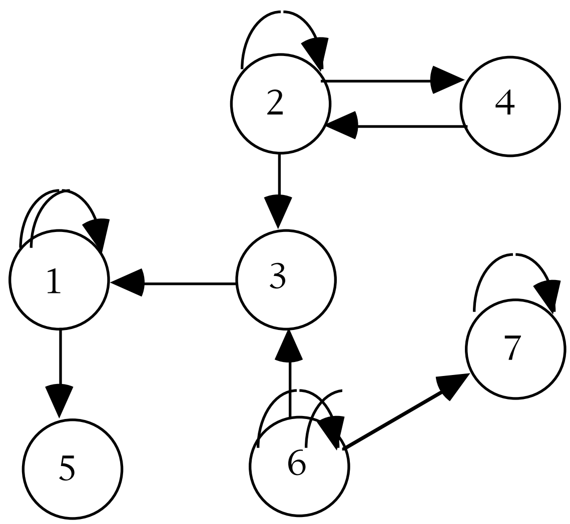
Now, some definitions:

1) $R \subseteq A \times A$ is **reflexive** if $(a,a) \in R$ for each $a \in A$.
(pictorially, loop from each node to itself)
2) $R \subseteq A \times A$ is **symmetric** if $(a,b) \in R$ whenever $(b,a) \in R$
(pictorially, a loop between pairs of nodes)
3) $R \subseteq A \times A$ is **antisymmetric** if whenever $(a,b) \in R$, $(b,a) \notin R$, for $a \neq b$.
(pictorially, no loops between pairs)
4) $R \subseteq A \times A$ is **transitive** if whenever $(a,b) \in R$ and $(b,c) \in R$, then $(a,c) \in R$.

An **equivalence relation** is one that is reflexive, symmetric, and transitive.

(reflexive, antisymmetric, and transitive is a partial order)

Example.



Reflexive? NO
Symmetric? NO
Transitive? NO