

Type Inference Against Races

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Abstract. The race condition checker `rccjava` uses a formal type system to statically identify potential race conditions in concurrent Java programs, but it requires programmer-supplied type annotations. This paper describes a type inference algorithm for `rccjava`. Due to the interaction of parameterized classes and dependent types, this type inference problem is NP-complete. This complexity result motivates our new approach to type inference, which is via reduction to propositional satisfiability. This paper describes our type inference algorithm and its performance on programs of up to 30,000 lines of code.

1 Introduction

A race condition occurs when two threads in a concurrent program manipulate a shared data structure simultaneously, without synchronization. Errors caused by race conditions are notoriously hard to catch using testing because they are scheduling dependent and difficult to reproduce. Typically, programmers attempt to avoid race conditions by adopting a programming discipline in which shared variables are protected by locks.

In a previous paper [10], we described a static analysis tool called `rccjava` that enforces this lock-based synchronization discipline. The analysis performed by `rccjava` is formalized as a type system, and it incorporates features such as *dependent types* (where the type of a field describes the lock protecting it) and *parameterized classes* (where fields in different instances of a class can be protected by different locks).

Our previous evaluation of `rccjava` indicates that it is effective for catching race conditions. However, `rccjava` relies on programmer-inserted type annotations that describe the locking discipline, such as which lock protects a particular field. The need for these type annotations limits `rccjava`'s applicability to large, legacy systems. Hence, to achieve practical static race detection for large programs, annotation inference techniques are necessary.

In previous work along these lines, we developed Houdini/rcc [11], a type inference algorithm for `rccjava` that heuristically generates a large set of candidate type annotations and then iteratively removes all invalid annotations. However, this approach could not handle parameterized classes or methods, which limits its ability to handle many of the synchronization idioms of real programs.

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In the presence of parameterized classes, the type inference problem for `rccjava` is NP-complete, meaning that any type inference algorithm will have an exponential worst-case behavior. This complexity result motivates our new approach to type inference, which is via reduction to propositional satisfiability. That is, given an unannotated (or partially-annotated) program, we translate this program into a propositional formula that is satisfiable if and only if the original program is typeable. Moreover, after computing a satisfying assignment for the generated formula, we translate this assignment into appropriate annotations for the program, yielding a valid, explicitly-typed program. This approach works well in practice, and we report on its performance on programs of up to 30,000 lines of code.

Producing a small number of meaningful error messages for erroneous or untypeable programs is often challenging. We tackle this aspect of type inference by generating a weighted MAX-SAT problem [4] and producing error messages for the unsatisfied clauses in the optimal solution. Our experience shows that the resulting warnings often correspond to errors in the original program, such as accessing a field without holding the appropriate lock.

We have implemented our algorithm in the *Rcc/Sat* tool for multithreaded Java programs. Experiments on benchmark programs demonstrate that it is effective at inferring valid type annotations for multithreaded code. The algorithm’s precision is significantly improved by performing a number of standard analyses, such as control-flow and escape analysis, prior to type checking.

The key contributions of this paper include:

- a type inference algorithm based on reduction to propositional satisfiability;
- a refinement of this approach to generate useful error messages via reduction to weighted MAX-SAT; and
- experimental results that validate the effectiveness of this approach.

The annotations constructed by *Rcc/Sat* also provide valuable documentation to the programmer; facilitate checking other properties such as atomicity [16, 15, 12]; and can help reduce state explosion in model checkers [24, 25, 14, 9].

2 Types Against Races

2.1 Type Checking

This section introduces RFJ2, an idealized multithreaded subset of Java with a type system that guarantees race freedom for well-typed programs. This type system extends our previous work on the `rccjava` type system [10], for example with parameterized methods. To clarify our presentation, RFJ2 also simplifies some aspects of `rccjava`. For example, it does not support inheritance. (Inheritance and other aspects of the full Java programming language are dealt with in our implementation, described in Section 4.)

An RFJ2 program (see Figure 1) is a sequence of class declarations together with an initial expression. Each class declaration associates a class name with

$P ::= \text{defn}^* e$	(program)	
$\text{defn} ::= \text{class } cn \langle \text{ghost } x^* \rangle \{ \text{field}^* \text{ meth}^* \}$	(class declaration)	
$\text{field} ::= t \text{ fn } \text{guarded_by } l$	(field declaration)	
$\text{meth} ::= t \text{ mn} \langle \text{ghost } x^* \rangle (\text{arg}^*) \text{ requires } s \{ e \}$	(method declaration)	
$\text{arg} ::= t x$	(argument declaration)	
$c, t ::= cn \langle l^* \rangle$	(type)	
$l ::= x \mid \alpha \mid l \cdot \theta$	(lock expression)	
$s ::= \emptyset \mid \{l\} \mid s \cup s \mid \beta \mid s \cdot \theta$	(lock set expression)	
$\theta ::= [x_1 := l_1, \dots, x_n := l_n]$	(substitution)	
$e, f ::= x \mid \text{null} \mid \text{new } c(e^*) \mid e.\text{fn} \mid e.\text{fn} = e \mid e.\text{mn} \langle l^* \rangle (e^*)$	(expressions)	
$\mid \text{let } x = e \text{ in } e \mid \text{synchronized } x e \mid e.\text{fork}$		
$\alpha \in \text{LockVar}$	$x, y \in \text{Var}$	$fn \in \text{FieldName}$
$\beta \in \text{LockSetVar}$	$cn \in \text{ClassName}$	$mn \in \text{MethodName}$

Fig. 1. The idealized language RFJ2.

a body that consists of a sequence of field and method declarations. The self-reference variable “**this**” is implicitly bound within the class body.

The RFJ2 language includes type annotations that specify the locking discipline. For example, the type annotation **guarded_by** x on a field declaration states that the lock denoted by the variable x must be held whenever that field is accessed (read or written). Similarly, the type annotation **requires** x_1, \dots, x_n on a method declaration states that these locks are held on method entry; the type system verifies that these locks are indeed held at each call-site of the method, and checks that the method body is race-free given this assumption.

The language provides *parameterized classes*, to allow the fields of a class to be protected by some lock external to the class. A parameterized class declaration

$$\text{class } cn \langle \text{ghost } x_1 \dots x_n \rangle \{ \dots \}$$

introduces a binding for the *ghost* variables $x_1 \dots x_n$, which can be referred to from type annotations within the class body. The type $cn \langle y_1 \dots y_n \rangle$ refers to an *instantiated version* of cn , where each x_i in the body is replaced by y_i . As an example, the type $\text{Hashtable} \langle y_1, y_2 \rangle$ may denote a hashtable that is protected by lock y_1 , where each element of the hashtable is protected by lock y_2 .

The RFJ2 language also supports *parameterized method* declarations, such as

$$t \text{ m} \langle \text{ghost } x \rangle (cn \langle x \rangle y) \text{ requires } x \{ \dots \}$$

which defines a method m that is parameterized by lock x , and which takes an argument of type $cn \langle x \rangle$. A corresponding invocation $e.m \langle z \rangle (e')$ must supply a ghost argument z and an actual parameter e' of type $cn \langle z \rangle$.

Expressions include object allocation **new** $c(e^*)$, which initializes a new object’s fields with its argument values; field read and update; method invocation;

(a) Example Program Ref

<pre> class Lock() { } class Ref(ghost x) { int y guarded_by α_1 boolean lessThan(Ref(α_2) o) requires β { this.y < o.y ; } } </pre>	<pre> let lock = new Lock()(); r1 = new Ref(α_3)(1); r2 = new Ref(α_4)(2) in synchronized (lock) { r1.lessThan(r2); } </pre>
--	--

(b) Constraints

$\alpha_1 \in \{ \text{this}, x \}$	decl. of y
$\alpha_2 \in \{ \text{this}, x \}$	decl. of lessThan
$\beta \subseteq \{ \text{this}, x, o \}$	decl. of lessThan
$\alpha_3 \in \{ \text{lock} \}$	first new expr.
$\alpha_4 \in \{ \text{lock}, r1 \}$	second new expr.
$\alpha_1 \in \beta$ access to this.y $\alpha_1[\text{this} := o, x := \alpha_2] \in \beta$ access to o.y $\beta[\text{this} := r1, x := \alpha_3, o := r2] \subseteq \{ \text{lock} \}$ requires for call $\alpha_2[\text{this} := r1, x := \alpha_3, o := r2] = \alpha_4$ arg. type for call	

(c) Conditional Assignment

$Y(\alpha_1) = (b_1 ? \text{this} : x)$	decl. of y
$Y(\alpha_2) = (b_2 ? \text{this} : x)$	decl. of lessThan
$Y(\beta) = (b_4 ? \text{this} : \emptyset) \cup (b_5 ? x : \emptyset) \cup (b_6 ? o : \emptyset)$	decl. of lessThan
$Y(\alpha_3) = \text{lock}$	first new expr.
$Y(\alpha_4) = (b_3 ? \text{lock} : r1)$	second new expr.

(d) Boolean Constraints

$(b_1 ? \text{this} : x) \in (b_4 ? \text{this} : \emptyset) \cup (b_5 ? x : \emptyset) \cup (b_6 ? o : \emptyset)$	access to this.y
$(b_1 ? o : (b_2 ? \text{this} : x)) \in (b_4 ? \text{this} : \emptyset) \cup (b_5 ? x : \emptyset) \cup (b_6 ? o : \emptyset)$	access to o.y
$(b_4 ? r1 : \emptyset) \cup (b_5 ? \text{lock} : \emptyset) \cup (b_6 ? r2 : \emptyset) \subseteq \{ \text{lock} \}$	requires for call
$(b_2 ? r1 : \text{lock}) = (b_3 ? \text{lock} : r1)$	arg. type for call

(e) Boolean Formula

$[(b_1 \wedge b_4) \vee (\neg b_1 \wedge b_5)]$	access to this.y
$\wedge [(b_1 \wedge b_6) \vee (\neg b_1 \wedge ((b_2 \wedge b_4) \vee (\neg b_2 \wedge b_5)))]$	access to o.y
$\wedge [\neg b_4 \wedge \neg b_6]$	requires for call
$\wedge [(b_2 \wedge \neg b_3) \vee (\neg b_2 \wedge b_3)]$	arg. type for call

Fig. 2. Example program and type inference constraints.

and variable binding and reference. The expression `synchronized x e` is evaluated in a manner similar to Java's `synchronized` statement: the lock for object `x` is acquired, the subexpression `e` is then evaluated, and finally the lock is released. The expression `e.fork` starts a new thread. Here, `e` should evaluate to an object that includes a nullary method `run`. The fork operation spawns a new thread that calls that `run` method.

The RFJ2 type system leverages parameterized methods to reason about thread-local data. (This approach replaces the escape by analysis embedded in our earlier type system [10].) Specifically, the `run` method of each forked thread takes a `ghost` parameter `tl_lock` denoting a *thread-local* lock that is always held

by that thread:

```
t run(ghost t1.lock)() requires t1.lock { e }
```

Intuitively, the underlying run-time system creates and acquires this thread-local lock when a new thread is created. This lock may be used to guard thread-local data and may be passed as a ghost parameter to other methods that access thread-local data. In a similar fashion, we also introduce an implicit, globally-visible lock called `main_lock`, which is held by the initial program thread and can be used to protect data exclusively accessed by that thread.

2.2 Type Inference

Our previous evaluation of the race-free type system `rccjava` indicates that it is effective for catching race conditions [10]. However, the need for programmer-inserted annotations limits its applicability to large, legacy systems, which motivates the development of type inference techniques for race-free type systems.

In this paper we describe a novel type inference system for RFJ2. We introduce *lock variables* α and *lockset variables* β , collectively referred to as *locking variables*. Locking variables may be mentioned in type annotations, as in `guarded_by` α , `requires` β , or `cn`(α_1, α_2). During type inference, each lock variable α is resolved to some specific program variable in scope, and each lock set variable β is resolved to some set of program variables in scope. As an example, Figure 2(a) presents a simple reference cell implementation, written in RFJ2 extended with primitive types and operations, that contains locking variables.

An RFJ2 program is *explicitly-typed* if it contains no locking variables. The *type inference* problem is, given a program with locking variables, to resolve these locking variables so that the resulting explicitly-typed program is well-typed.

Parameterized classes introduce substitutions that complicate the type inference problem. We use the notation $[x_1 := l_1, \dots, x_n := l_n]$ to denote a substitution θ that replaces each program variable x_i with the lock expression l_i . To illustrate the need for these substitutions, consider the class declaration:

```
class cn(ghost x) { t fn guarded_by l; }
```

If a variable p has type `cn`(y), then the field `p.fn` is protected by $\theta(l)$, where the substitution $\theta \equiv [x := y]$ replaces the formal ghost parameter x by the actual parameter y . The application of a substitution to most syntactic entities is straightforward; however, the application of a substitution θ to a lock expression l is delayed until any lock variables α in the lock expression are resolved. We use the syntax $l \cdot \theta$ to represent this *delayed substitution*. Similarly, if the *lock set expression* s denote the set of locks in a method's `requires` clause, then the application of a substitution θ to s yields the delayed substitution $s \cdot \theta$. The following examples illustrate substitutions on various syntactic entities. (Due to space limitations, we do not present an exhaustive definition.)

$$\begin{aligned} \theta(x) &= l \quad \text{if } \theta \equiv [\dots, x := l, \dots] \\ \theta(\text{synchronized } x \ e) &= \text{synchronized } \theta(x) \ \theta(e) \\ \theta(l) &= l \cdot \theta \\ \theta(s) &= s \cdot \theta \end{aligned}$$

Since the type rules reason about delayed substitutions, we include these delayed substitutions in the programming language syntax, but we require that substitutions do not appear in source programs.

The type rules for RFJ2 generate a collection of *constraints* that contain delayed substitutions. These constraints include equality constraints between lock expressions and containment constraints between lock set expressions:

$$C ::= s \subseteq s \mid l = l$$

The core of the type system is defined by the judgment:

$$P; E; s \vdash e : t \ \& \ \bar{C}$$

Here, the program P is included to provide access to class declarations; E is an environment providing types for the free variables of the expression e ; the lock set s describes the locks held when executing e ; t is the type inferred for e ; and \bar{C} is the generated set of constraints.

Most of the type rules are straightforward. The complete set of type judgments and rules is contained in Appendix A. Here we briefly explain two of the more crucial rules. The rule for `synchronized` $x e$ checks e with an extended lock set that includes x , since the lock x is held when evaluating e . The rule for $e.fn$ checks that e is a well-typed expression of some class type $cn\langle l_{1..n} \rangle$ and that cn has a field fn of type t , guarded by lock l .

$$\frac{\frac{P; E; s \vdash x : t' \ \& \ \bar{C}}{P; E; s \cup \{x\} \vdash e : t \ \& \ \bar{C}'}}{P; E; s \vdash \text{synchronized } x e : t \ \& \ (\bar{C} \cup \bar{C}')}}{\frac{\frac{P; E; s \vdash e : cn\langle l_{1..n} \rangle \ \& \ \bar{C}}{\text{class } cn(\text{ghost } x_{1..n}) \{ \dots t \text{ fn guarded_by } l \dots \} \in P}}{\theta = [\text{this} := e, x_j := l_j \text{ } j \in 1..n]} \quad \frac{P; E \vdash \theta(t) \ \& \ \bar{C}'}{P; E; s \vdash e.fn : \theta(t) \ \& \ (\bar{C} \cup \bar{C}' \cup \{\theta(l) \in s\})}}$$

Since the protecting lock expression l (and type t) may refer to the ghost parameters $x_{1..n}$ and the implicitly-bound self-reference `this`, neither of which are in scope at the field access, we introduce the substitution θ which substitutes appropriate expressions for these variables. The constraint $\theta(l) \in s$, an abbreviation for $\{\theta(l)\} \subseteq s$, ensures that the substituted lock expression is in the current lock set. The type of the field dereference is computed by applying the substitution θ to the field type t , which must yield a well-formed type.

The type system defines the top-level judgment $P \vdash \bar{C}$, where \bar{C} is the generated set of constraints for the program P . Applying these type rules to the example program `Ref` of Figure 2(a) yields the constraints shown in Figure 2(b). (We ignore `main_lock` in this example for simplicity).

We next address the question of when the generated constraints over the locking variables are satisfiable. An *assignment*

$$A : (\text{LockVar} \rightarrow \text{Var}) \cup (\text{LockSetVar} \rightarrow 2^{\text{Var}})$$

resolves lock and lock set variables to corresponding program variables and sets of program variables, respectively. We extend assignments to lock expressions, lock

set expressions, and substitutions. In particular, since an assignment resolves all locking variables, any delayed substitutions can be immediately performed.

$$\begin{array}{lll}
A : l \rightarrow \text{Var} & A : s \rightarrow 2^{\text{Var}} & A : \theta \rightarrow \theta \\
A(x) = x & A(\emptyset) = \emptyset & A([x_1 := l_1, \dots, x_n := l_n]) = \\
A(l \cdot \theta) = A(\theta)(A(l)) & A(\{l\}) = \{A(l)\} & [x_1 := A(l_1), \dots, x_n := A(l_n)] \\
A(s_1 \cup s_2) = A(s_1) \cup A(s_2) & & \\
A(s \cdot \theta) = A(\theta)(A(s)) & &
\end{array}$$

We also extend assignments in a compatible manner to other syntactic units, such as constraints, expressions, programs, etc.

An assignment A *satisfies* a constraint C (written $A \models C$) as follows:

$$\begin{array}{ll}
A \models s_1 \subseteq s_2 & \text{iff } A(s_1) \subseteq A(s_2) \\
A \models l_1 = l_2 & \text{iff } A(l_1) = A(l_2)
\end{array}$$

If $A \models C$ for all $C \in \bar{C}$ then we say A is a *solution* for \bar{C} , written $A \models \bar{C}$. A set of constraints \bar{C} is *valid*, written $\models \bar{C}$, if every assignment is a solution for \bar{C} . For example, the constraints of Figure 2(b) for the program `Ref` are satisfied by the assignment: $\alpha_1 = \alpha_2 = \mathbf{x}$, $\alpha_3 = \alpha_4 = \text{lock}$, and $\beta = \{\mathbf{x}\}$.

We say P is *well-typed* if $P \vdash \bar{C}$ and the constraints \bar{C} are satisfiable. If a solution A for the constraints \bar{C} exists, the following theorem states that the explicitly-typed program $A(P)$ is well-typed. (Proofs for the theorems in this paper appear in an extended report [13].)

Theorem 1. *If $P \vdash \bar{C}$ and $A \models \bar{C}$ then $A(P) \vdash A(\bar{C})$ and $\models A(\bar{C})$.*

For explicitly-typed programs, since the generated constraints \bar{C} do not contain locking variables, checking the satisfiability of \bar{C} is straightforward. In the more general case where P is not explicitly-typed, the type inference problem involves *searching* for a solution A for the generated constraints \bar{C} . Due to the interaction between parameterized classes and dependent types, the type inference problem for `RFJ2` (and similarly for `rccjava`) is NP-complete. (The proof is via a reduction from propositional satisfiability.)

Theorem 2. *For an arbitrary program P , the problem of finding an assignment A such that $A(P)$ is explicitly-typed and $A(P) \vdash \bar{C}$ and $\models \bar{C}$ is NP-complete.*

Despite this worst-case complexity result, we demonstrate a technique in the next section that has proven effective in practice.

3 Solving Constraint Systems

3.1 Generating Boolean Constraints

For each lock variable α mentioned in the program, the type rules introduce a *scope constraint* $\alpha \in \{x_1, \dots, x_n\}$ that constrains α to be one of the variables x_1, \dots, x_n in scope. A similar constraint $\beta \subseteq \{x_1, \dots, x_n\}$ is introduced for

each lock set variable β . These scope constraints specify the possible choices for each locking variable, and enable us to translate each constraint C over locking variables into a *Boolean constraint* D that uses Boolean variables to encode the possible choices for each locking variable. The notation $b?X : Y$ denotes X if the Boolean variable b is true, and denotes Y otherwise.

$$\begin{array}{ll}
D ::= S \subseteq S \mid L = L & \text{(Boolean constraints)} \\
L ::= x \mid b?L : L & \text{(conditional lock expressions)} \\
S ::= \emptyset \mid \{L\} \mid b?S : S \mid S \cup S & \text{(conditional lock set expressions)} \\
b \in \text{BoolVar} & \text{(Boolean variables)}
\end{array}$$

From the scope constraints, we generate a *conditional assignment*

$$Y : (\text{LockVar} \rightarrow L) \cup (\text{LockSetVar} \rightarrow S)$$

that encodes the possible choices for each locking variable. For example, the scope constraints $\alpha \in \{x_1, \dots, x_n\}$ and $\beta \subseteq \{y_1, \dots, y_m\}$ yield:

$$\begin{aligned}
Y(\alpha) &= b_1?x_1 : (b_2?x_2 : (\dots b_{n-1}?x_{n-1} : x_n) \dots) \\
Y(\beta) &= (b'_1?\{y_1\} : \emptyset) \cup \dots \cup (b'_m?\{y_m\} : \emptyset)
\end{aligned}$$

where each Boolean variable b_i and b'_i is fresh.³

We extend the conditional assignment to translate each constraint C to a Boolean constraint $D = Y(C)$, and to translate lock expressions, lock set expressions, and substitutions, as follows. Since the conditional assignment (conditionally) resolves locking variables, as part of this translation we immediately apply any delayed substitutions, to yield a substitution-free Boolean constraint:

$$\begin{array}{ll}
Y : C \rightarrow D & Y : s \rightarrow S \\
Y(s_1 \subseteq s_2) = Y(s_1) \subseteq Y(s_2) & Y(\emptyset) = \emptyset \\
Y(l_1 = l_2) = Y(l_1) = Y(l_2) & Y(\{l\}) = \{Y(l)\} \\
& Y(s_1 \cup s_2) = Y(s_1) \cup Y(s_2) \\
& Y(s \cdot \theta) = Y(\theta)(Y(s)) \\
\\
Y : l \rightarrow L & \\
Y(x) = x & Y([x_1 := l_1, \dots, x_n := l_n]) = \\
Y(l \cdot \theta) = Y(\theta)(Y(l)) & [x_1 := Y(l_1), \dots, x_n := Y(l_n)]
\end{array}$$

Figure 2(c) and (d) show the conditional assignment and Boolean constraints for the example program **Ref**.

A *truth assignment* $B : \text{BoolVar} \rightarrow \text{Boolean}$ assigns truth values to Boolean variables. We extend truth assignments to L and S in a straightforward manner:

$$\begin{array}{ll}
B : L \rightarrow \text{Var} & B : S \rightarrow 2^{\text{Var}} \\
B(x) = x & B(\emptyset) = \emptyset \\
B(b?L_1 : L_2) = \begin{cases} B(L_1) & \text{if } B(b) \\ B(L_2) & \text{if } \neg B(b) \end{cases} & B(\{L\}) = \{B(L)\} \\
& B(b?S_1 : S_2) = \begin{cases} B(S_1) & \text{if } B(b) \\ B(S_2) & \text{if } \neg B(b) \end{cases} \\
& B(S_1 \cup S_2) = B(S_1) \cup B(S_2)
\end{array}$$

³ We could encode the choice for the first constraint as a decision tree with only $\log n$ Boolean variables.

A truth assignment B satisfies a set of Boolean constraints \bar{D} if $B \models D$ for each $D \in \bar{D}$, where:

$$\begin{aligned} B \models S_1 \subseteq S_2 & \text{ iff } B(S_1) \subseteq B(S_2) \\ B \models L_1 = L_2 & \text{ iff } B(L_1) = B(L_2) \end{aligned}$$

For example, the Boolean constraints of Figure 2(d) are satisfied by the following truth assignment: $b_1 = b_2 = b_4 = b_6 = \mathbf{false}$ and $b_3 = b_5 = \mathbf{true}$.

The application of a truth assignment B to a conditional assignment Y yields the (unconditional) assignment $B(Y)$, defined as $B(Y)(x) = B(Y(x))$.

The translation from constraints to Boolean constraints is semantics-preserving, in the sense that if the generated Boolean constraints are satisfiable, then the original constraints are also satisfiable.

Theorem 3. *Suppose $\bar{D} = Y(\bar{C})$ and let B be a truth assignment. Then $B(Y) \models \bar{C}$ if and only if $B \models \bar{D}$.*

3.2 Solving Boolean Constraints

The final step is to find a truth assignment B satisfying the generated Boolean constraints \bar{D} . We accomplish this step by translating \bar{D} into a Boolean formula F , which can then be solved by a standard propositional satisfiability solver such as Chaff [21]. The Boolean formula syntax and this translation are as follows:

$$\begin{aligned} F & ::= \mathbf{true} \mid \mathbf{false} \mid b \mid F \vee F \mid F \wedge F \mid \neg F \\ & \llbracket \cdot \rrbracket : \bar{D} \rightarrow F \\ & \llbracket \bar{D} \rrbracket = \bigwedge_{D \in \bar{D}} \llbracket D \rrbracket \\ & \llbracket \cdot \rrbracket : D \rightarrow F \\ \llbracket x = x \rrbracket &= \mathbf{true} & \llbracket (b?S_1 : S_2) \subseteq S \rrbracket &= (b \wedge \llbracket S_1 \subseteq S \rrbracket) \\ \llbracket x = y \rrbracket &= \mathbf{false} \text{ if } x \neq y & & \vee (\neg b \wedge \llbracket S_2 \subseteq S \rrbracket) \\ \llbracket L = (b?L_1 : L_2) \rrbracket &= (b \wedge \llbracket L = L_1 \rrbracket) \\ & \vee (\neg b \wedge \llbracket L = L_2 \rrbracket) & \llbracket \{L\} \subseteq \emptyset \rrbracket &= \mathbf{false} \\ \llbracket (b?L_1 : L_2) = L \rrbracket &= \llbracket L = (b?L_1 : L_2) \rrbracket & \llbracket \{L\} \subseteq (b?S_1 : S_2) \rrbracket &= (b \wedge \llbracket \{L\} \subseteq S_1 \rrbracket) \\ & & & \vee (\neg b \wedge \llbracket \{L\} \subseteq S_2 \rrbracket) \\ \llbracket \emptyset \subseteq S \rrbracket &= \mathbf{true} & \llbracket \{L\} \subseteq (S_1 \cup S_2) \rrbracket &= \llbracket \{L\} \subseteq S_1 \rrbracket \\ \llbracket (S_1 \cup S_2) \subseteq S \rrbracket &= \llbracket S_1 \subseteq S \rrbracket \\ & \wedge \llbracket S_2 \subseteq S \rrbracket & & \vee \llbracket \{L\} \subseteq S_2 \rrbracket \\ & & \llbracket \{L_1\} \subseteq \{L_2\} \rrbracket &= \llbracket L_1 = L_2 \rrbracket \end{aligned}$$

Figure 2(e) presents the formulas for the four constraints from our example program. This translation is semantics preserving with respect to the standard notion of satisfiability $B \models F$ for Boolean formulas.

Theorem 4. *If $F = \llbracket \bar{D} \rrbracket$ then for all B , $B \models F$ if and only if $B \models \bar{D}$.*

In summary, our type inference algorithm proceeds as follows: Given a program P with locking variables, we generate from P a collection of constraints \bar{C} over the locking variables; we extract a conditional assignment Y from \bar{C} and generate Boolean constraints $\bar{D} = Y(\bar{C})$; and we generate a corresponding Boolean formula $F = \llbracket \bar{D} \rrbracket$. We use a propositional satisfiability solver to determine a truth assignment B for F , in which case we also have that $B \models \bar{D}$

by Theorem 4 and $(B(Y)) \models \bar{C}$ by Theorem 3, and therefore the explicitly-typed program $(B(Y))(P)$ is well-typed. Conversely, if the generated formula F is unsatisfiable, then there is no assignment A such that $A(P)$ is well-typed.

4 Implementation

We have implemented our inference algorithm in the Rcc/Sat checker, which supports the full Java programming language (although it does not currently detect race conditions on array accesses). Rcc/Sat takes as input an unannotated or partially-annotated program, where any typing annotations are provided in comments starting with “#”, as in `/*# guarded_by y */`.

Rcc/Sat first adds a predetermined number of `ghost` parameters to all classes and methods lacking user-specified parameters. Next, for each unguarded field, Rcc/Sat adds the annotation `guarded_by` α , where α is fresh. Rcc/Sat also uses fresh locking variables to add any missing `requires` annotations and class and method instantiation parameters. Rcc/Sat then performs our type inference algorithm. If the generated constraints are satisfiable, then the satisfying assignment is used to generate an explicitly-typed version of the program. Section 4.2 outlines how we generate meaningful error messages when they are not.

4.1 Java Features

We handle additional features of the Java programming language as follows.

Scope constraints. Rcc/Sat permits lock expressions to be any *final* object references, including: (1) `this`; (2) ghost parameters; (3) `final` variables, static fields, and parameters; and (4) well-typed expressions of the form $e.f$, where e is a constant expression and f is a `final` field. This set may be infinite, and we heuristically limit it to expressions with at most two field accesses.

Inheritance, subtyping, and interfaces. Given the declaration

```
class C<ghost a1, ..., an> extends D<ghost b1, ..., bk> { ... }
```

we consider the type instantiation $C\langle l_{1..n} \rangle$ to be an immediate subtype of $D\langle m_{1..k} \rangle$ provided $m_i \equiv b_i[a_j := l_j \text{ } j \in 1..n]$ for all $i \in 1..k$. The subtyping relation is the reflexive and transitive closure of this rule. The signature of an overriding method must match that of the overridden form, after applying the type parameter substitutions induced by the inheritance hierarchy. Interfaces are handled similarly.

Inner classes. Non-static inner classes may access the type parameters from the enclosing class and may declare their own parameters. Thus, the complete type for such a class is `Outer` $\langle l_{1..n} \rangle$.`Inner` $\langle m_{1..k} \rangle$.

Static fields, methods, and inner classes. Static members may not refer to the enclosing class’ type parameters since static members are not associated with a specific instantiation of the class.

Thread objects. To allow `Thread` objects to store thread-local data in their fields, Rcc/Sat adds an implicit `final` field `tl_lock` to each `Thread` class. This field is analogous to (and replaces) the `ghost` parameter on the `run` method in RfJ2. It may guard other fields and is assumed to be held when `run` is invoked.

Escape mechanisms. We provide escapes from the RFJ2 type system through a “no_warn” annotation that suppresses the generation of constraints for a line of code. Also, since ghost parameters are erased at run time, the ghost parameters in typecasts of the form $C\langle a \rangle x$ are not checked dynamically.

4.2 Reporting Errors

We introduce two important improvements that enable the tool to pinpoint likely errors in the program when the generated constraints are unsatisfiable.

First, we change the algorithm to check each field declaration in a program separately, thereby enabling us to distinguish fields with potential races from those that are race-free. To check a single field, we generate the constraints as before, except that we only add field access constraints for accesses to the field of interest. The analysis is compositional in this manner because the presence or absence of races on one field is independence of races on other fields.

There is a possibility that the same locking variable will be assigned different values when checking different fields. If this occurs, we can compose the results of the separate checks together by introducing additional type parameters and renaming locking variables as necessary. For example, if a type instantiation $C\langle \alpha \rangle$ of class $C\langle \text{ghost } x \rangle$ becomes $C\langle 11 \rangle$ when checking one field of C and $C\langle 12 \rangle$ when checking another, we can change the class declaration to $C\langle \text{ghost } x1, x2 \rangle$, and instantiate it as $C\langle 11, 12 \rangle$ at the conflicting location.

Second, when there are race conditions on a field, it is often desirable to infer the most likely lock protecting it and then generate errors for locations where that lock is not held. For example, the following program is not well-typed:

```

1: class C(ghost y) {
2:   int c guarded_by  $\alpha$ ;
3:   void f1() requires y { c = 1; }
4:   void f2() requires y { c = 2; }
5:   void f3() requires this { c = 3; }
6: }
```

Our tool produces the following diagnostic message at the likely error site:

```
C.java:5: Lock 'y' not held on access to 'c'. Locks held: { this }.
```

To pinpoint likely error locations in this way, we express type inference as an optimization problem instead of a satisfiability problem. First, we add weights to some of the generated constraints, as follows. A constraint C with weight w is written as the weighted constraint $W = C|_w$.

$\alpha \in \{\text{y, this, no_lock}\}$	Scope constraint for c
$\alpha \in \{\text{y, this}\} _2$	Requirement that c is guarded by a valid lock
$\alpha \in \{\text{y, no_lock}\} _1$	Access constraint for c from $f1$
$\alpha \in \{\text{y, no_lock}\} _1$	Access constraint for c from $f2$
$\alpha \in \{\text{this, no_lock}\} _1$	Access constraint for c from $f3$

These five constraints refer to `no_lock`, a lock name used in the checker to indicate that no reasonable guarding lock can be found for a field. Given constraints \bar{C} and weighted constraints \bar{W} , we compute the optimal assignment A such that:

1. $A \models C$ for all $C \in \bar{C}$, and
2. the sum $\sum \{w \mid C|_w \in \bar{W} \wedge A \models C\}$ is maximized.

Note that we do not require all constraints in \bar{W} be satisfied by A . For the constraints above, A is the assignment $\alpha = \mathbf{y}$, with a value of 4. We then generate error messages for all constraints in \bar{W} that are not satisfied by A . The constraint $\alpha \in \{\mathbf{this}, \mathbf{no_lock}\}_1$ is not satisfied by the optimal assignment A , yielding the above error message. Conversely, if the optimal assignment A did not satisfy the constraint $\alpha \in \{\mathbf{y}, \mathbf{this}\}_2$, then we would generate the error message:

```
C.java:2: No consistent guarding lock for field 'c'.
```

We have found that the heuristic of weighting declaration constraints 2–4 times more than field access constraints works well in practice.

We solve the constraint optimization problem for \bar{W} and \bar{C} by translating the constraints into a weighted MAX-SAT problem and solving it with the PBS tool [4]. The translation is similar to the case without weights. PBS and similar tools can find optimal assignments for formulas including up to 50–100 weighted clauses. Optimizing over a larger number of weighted clauses is currently computationally intractable. Thus, we still check one field at a time and only optimize over constraints generated by field accesses, placing all constraints for `requires` clauses and type equality in \bar{C} . If \bar{C} is not satisfiable, we forego the optimization step and instead generate error messages for constraints in the smallest unsatisfiable core of \bar{C} , which we find with Chaff [21].

4.3 Improving Precision

Rcc/Sat implements a somewhat more expressive type system than that described in Section 2 to handle the synchronization patterns of large programs more effectively. In particular:

- Unreachable code is not type checked.
- Read-shared fields do not need guarding locks. A *read-shared* field is a field that is initialized while local to its creating thread, and subsequently shared in read-only mode among multiple threads.
- A field’s protecting lock need not be held for accesses occurring when only a single thread exists or when the object has not escaped its creating thread.

Programs typically relax the core lock-based synchronization discipline along these lines. The checker currently uses quite basic implementations of rapid type analysis [5], escape analysis [6], and control-flow analysis for this step. Using more precise analyses would further improve our type inference algorithm.

5 Evaluation

We applied Rcc/Sat to benchmark programs including `elevator`, a discrete event simulator [28]; `tsp`, a Traveling Salesman Problem solver [28]; `sor`, a scientific computing program [28]; the `mtrt` ray-tracing program and `jbb` business

Program	Size (LOC)	Time (s)	Time/ Field (s)	Number of Constraints	Formula Size		Manual Annot.	Fields			
					vars	clauses		Total	read- shared	race- free	no guard
elevator	529	5.0	0.22	215	1,449	3,831	0	23	17	6	0
tsp	723	6.9	0.19	233	2,090	7,151	3	37	21	16	3
sor	687	4.5	0.15	130	562	1,205	1	29	22	7	0
raytracer	1,982	21.0	0.27	801	9,436	29,841	2	77	45	28	4
moldyn	1,408	12.6	0.12	904	4,011	10,036	3	107	57	44	6
montecarlo	3,674	20.7	0.19	1,097	9,003	25,974	1	110	68	42	0
mtrt	11,315	138.8	1.5	5,636	38,025	123,046	6	181	112	69	4
jbb	30,519	2,773.5	3.52	11,698	146,390	549,667	40	787	472	295	20

Table 1. Summary of test program performance.

objects simulator benchmarks [23]; and the `moldyn`, `montecarlo`, and `raytracer` benchmarks [20]. We ran these experiments on a 3.06GHz Pentium 4 processor with 2GB of memory, with `Rcc/Sat` configured to insert one `ghost` parameter on classes, interfaces, and instance methods and two parameters on static methods.

Table 1 shows, for each benchmark, the size in lines of code, the overall time for type inference, and the average type inference time per field. It also shows the size of the constraint problem generated, in number of constraints and the number of variables and clauses in the resulting Boolean formula, after conversion to CNF. The preliminary analyses described in Section 4.3 typically consumed less than 2% of the run time on the larger benchmarks.

The “Manual Annotations” column reflects the number of annotations manually inserted to guide the analysis. We added these few annotations to suppress warnings only in situations where immediately identifiable local properties ensured correctness. The manual annotations were inserted, for example, to delineate single-threaded parts of the program after joining all spawned threads; to explicitly instantiate classes in two places where the scope constraint generation heuristics did not consider the appropriate locks; and to identify thread-local object references not found by our escape analysis. In `jbb`, we also added annotations to suppress spurious race-condition warnings on roughly 25 fields with benign races. These fields were designed to be *write-protected* [12], meaning that a lock guarded write accesses, but read accesses were not synchronized. This idiom is unsafe if misused but permits synchronization-free accessor methods.

The last four columns show the total number of fields in the program, as well as their breakdown into read-shared fields, race-free fields, and fields for which no guarding lock was inferred. The analyses described in Section 4.3 reduced the number of fields without valid guards by 20%–75%, a significant percentage.

`Rcc/Sat` identified three fields in the `tsp` benchmark on which there are intentional races. On `raytracer`, `Rcc/Sat` identified a previously known race on a checksum field and reported spurious warnings on three fields. It also identified a known race on a counter in `mtrt`. The remaining warnings were spurious and could be eliminated by additional annotations or, in some cases, by improving the precision of the additional analyses of Section 4.3.

Overall, these results are quite promising. Manually inserting a small number of annotations enables `Rcc/Sat` to verify that the vast majority (92%–100%) of

fields are race-free. These results show a substantial improvement over previous type inference algorithms for race-free type systems, such as Houdini/rcc.

6 Related Work

Boyapati and Rinard have defined a race-free type system with a notion of object ownership [7]. They include special owners to indicate thread-local data, thereby allowing a single class declaration to be used for both thread-local instances and shared instances, which motivated some of our refinements in RFJ2. They present an *intraprocedural* algorithm to infer ownership parameters for class instantiations within a method. This simpler intraprocedural context yields equality constraints over lock variables, which can be efficiently solved using union-find. We believe it may be possible to extend our interprocedural type inference algorithm to accommodate ownership types. Grossman has developed a race-free type system for Cyclone, a statically safe variant of C [18]. Cyclone has a number of additional features, such as existential quantification and singleton types, and it remains to be seen how our techniques would apply in this setting.

The `requires` annotations used in our type system essentially constrain the *effects* that the method may produce. Thus, we are performing a form of effect reconstruction [27, 26], but our dependent types are not amenable to traditional effect reconstruction techniques. Similarly, the constraints of our type system do not exhibit the monotonicity properties that facilitate the polynomial time solvers used in other constraint-based analyses (see, for example, Aiken’s survey [2]). Cardelli [8] was among the first to explore type checking for dependent types. Our dependent types are comparatively limited in expressive power, but the resulting type checking and type inference problems are decidable.

Eraser [22] is a tool for detecting race conditions in unannotated programs dynamically (though it may fail to detect certain errors because of insufficient test coverage). Agarwal and Stoller [1] present a dynamic type inference technique for the type system of Boyapati and Rinard. Their technique extracts locking information from a program trace and then performs a static analysis involving unique pointer analysis [3] and intraprocedural ownership inference [7] to construct annotations. These dynamic analyses complement our static approach, and it may be possible to leverage their results to facilitate type inference.

A common and significant problem with many type-inference techniques is the inability to construct meaningful error messages when inference fails (see, for example, [29, 30, 19]). An interesting contribution of our approach is that we view type inference as an optimization problem over a set of constraints that attempts to produce the most reasonable error messages for a program.

7 Conclusions

This paper contributes a new type inference algorithm for race-free type systems, which is based on reduction to propositional satisfiability. Our experimental results demonstrate that this approach works well in practice on benchmarks of up

to 30,000 lines of code. Extending and evaluating this approach on significantly larger benchmarks remains an issue for future work. We also demonstrate extensions to facilitate reliable error reporting. We believe the resulting annotations and race-free guarantee provided by our type inference system have a wide range of applications in the analysis, validation, and verification of multithreaded programs. In particular, they provide valuable documentation to the programmer, they facilitate checking other program properties such as atomicity, and they can help reduce state explosion in model checkers.

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A Type System

This appendix provides a complete definition of RFJ2. We first informally define a number of predicates. (See [17] for their precise definition.)

Predicate	Meaning
$ClassOnce(P)$	no class is declared twice in P
$FieldsOnce(P)$	no class contains two fields with the same name
$MethodsOncePerClass(P)$	no method name appears more than once per class

A typing environment is defined as: $E ::= \emptyset \mid E, t \ x \mid E, \text{ghost } x$

$P \vdash \bar{C}$

$$\frac{\text{ClassOnce}(P) \quad \text{FieldsOnce}(P) \quad \text{MethodsOncePerClass}(P) \quad P = \text{defn}_{1..n} \ e \quad P \vdash \text{defn}_i \ \& \ \bar{C}_i \quad \forall i \in 1..n}{P; \text{ghost } \text{main_lock}; \{\text{main_lock}\} \vdash e : t \ \& \ \bar{C}} \quad \frac{}{P \vdash \bar{C}_{1..n} \cup \bar{C}}$$

$P \vdash \text{defn} \ \& \ \bar{C}$

$$\frac{\text{garg}_i = \text{ghost } x_i \quad \forall i \in 1..n \quad E = \text{garg}_{1..n}, \text{cn}(x_{1..n}) \ \text{this} \quad P; E \vdash \text{field}_i \ \& \ \bar{C}_i \quad \forall i \in 1..j \quad P; E \vdash \text{meth}_i \ \& \ \bar{C}'_i \quad \forall i \in 1..k}{P \vdash \text{class } \text{cn}(\text{ghost } x_{1..n}) \ \{ \text{field}_{1..j} \ \text{meth}_{1..k} \} \ \& \ \bar{C}} \quad \frac{}{\bar{C} = \bar{C}_{1..j} \cup \bar{C}'_{1..k}}$$

$P; E \vdash \text{wf} \ \& \ \bar{C}$

$$\frac{}{P; \emptyset \vdash \text{wf} \ \& \ \emptyset} \quad \frac{P; E \vdash t \ \& \ \bar{C} \quad x \notin \text{dom}(E)}{P; E, t \ x \vdash \text{wf} \ \& \ \bar{C}} \quad \frac{P; E \vdash \text{wf} \ \& \ \bar{C} \quad x \notin \text{dom}(E)}{P; E, \text{ghost } x \vdash \text{wf} \ \& \ \bar{C}}$$

$P; E \vdash t \ \& \ \bar{C}$

$$\frac{P; E \vdash \text{wf} \ \& \ \bar{C} \quad \text{class } \text{cn}(\text{ghost } x_i^{i \in 1..n}) \ \dots \in P \quad \bar{C}' = \bar{C} \cup \{l_i \in \text{dom}(E) \mid i \in 1..n\}}{P; E \vdash \text{cn}(l_{1..n}) \ \& \ \bar{C}'}$$

$P; E \vdash \text{meth} \ \& \ \bar{C}$

$P; E \vdash \text{field} \ \& \ \bar{C}$

$$\frac{P; E \vdash t \ \& \ \bar{C} \quad \bar{C}' = \bar{C} \cup \{l \in \text{dom}(E)\}}{P; E \vdash t \ \text{fn } \text{guarded_by } l \ \& \ \bar{C}'}$$

$$\frac{\text{garg}_i = \text{ghost } x_i \quad \forall i \in 1..n \quad E' = E, \text{garg}_{1..n}, \text{arg}_{1..d} \quad P; E'; s \vdash e : t \ \& \ \bar{C} \quad \bar{C}' = \bar{C} \cup \{s \subseteq \text{dom}(E')\} \quad s \text{ is either } \{y_1, \dots, y_k\} \text{ or } \beta}{P; E \vdash t \ \text{mn}(\text{ghost } x_{1..n})(\text{arg}_{1..d}) \ \text{requires } s \ \{e\} \ \& \ \bar{C}'}$$

$P; E; s \vdash e : t \ \& \ \bar{C}$

$$\frac{P; E \vdash c \ \& \ \bar{C}}{P; E; s \vdash \text{null} : c \ \& \ \bar{C}} \quad \frac{P; E \vdash \text{wf} \ \& \ \bar{C} \quad E = E_1, t \ x, E_2}{P; E; s \vdash x : t \ \& \ \bar{C}} \quad \frac{P; E; s \vdash e : \text{cn}(l_{1..n}) \ \& \ \bar{C} \quad P; E \vdash \text{cn}(l'_{1..n}) \ \& \ \bar{C}' \quad \bar{C}'' = \bar{C} \cup \bar{C}' \cup \{l_i = l'_i \mid i \in 1..n\}}{P; E; s \vdash e : \text{cn}(l'_{1..n}) \ \& \ \bar{C}''}$$

$$\frac{\text{y is fresh} \quad \theta = [x_j := l_j \mid j \in 1..n, \text{this} := y] \quad P; E, \text{cn}(l_{1..n}) \ y; s \vdash e_i : \theta(t_i) \ \& \ \bar{C}_i \quad \forall i \in 1..k \quad \text{class } \text{cn}(\text{ghost } x_{1..n}) \ \{ \text{field}_{1..k} \ \text{meth}_{1..m} \} \in P \quad \text{field}_i = t_i \ \text{fn}_i \ \text{guarded_by } l'_i \quad \forall i \in 1..k \quad P; E \vdash \text{cn}(l_{1..n}) \ \& \ \bar{C}'}{\bar{C}'' = \bar{C}_{1..k} \cup \bar{C}' \cup \{l_i \in \text{dom}(E) \mid i \in 1..n\}} \quad \frac{}{P; E; s \vdash \text{new } \text{cn}(l_{1..n})(e_{1..k}) : \text{cn}(l_{1..n}) \ \& \ \bar{C}''}$$

$$\frac{P; E; s \vdash e : \text{cn}(l_{1..n}) \ \& \ \bar{C} \quad \text{class } \text{cn}(\text{ghost } x_{1..n}) \ \{ \dots t \ \text{fn } \text{guarded_by } l \dots \} \in P \quad \theta = [\text{this} := e, x_j := l_j \mid j \in 1..n] \quad P; E \vdash e' : \theta(t) \ \& \ \bar{C}'}{P; E; s \vdash e.\text{fn} = e' : \theta(t) \ \& \ (\bar{C} \cup \bar{C}' \cup \{\theta(l) \in s\})}$$

$$\frac{P; E; s \vdash e_1 : t_1 \ \& \ \bar{C}_1 \quad P; E, t \ x; s \vdash e_2 : t_2 \ \& \ \bar{C}_2 \quad \theta = [x := e_1] \quad P; E \vdash \theta(t_2) \ \& \ \bar{C}_3 \quad C = (\bar{C}_1 \cup \bar{C}_2 \cup \bar{C}_3)}{P; E; s \vdash \text{let } x = e_1 \ \text{in } e_2 : \theta(t_2) \ \& \ C}$$

$$\frac{P; E; s \vdash e : \text{cn}(l_{1..n}) \ \& \ \bar{C} \quad \text{class } \text{cn}(\text{ghost } x_{1..n}) \ \{ \dots t \ \text{mn}(\text{ghost } y_{1..k})(t_j \ z_j^{j \in 1..d}) \ \text{requires } s' \ \{e'\} \dots \} \in P \quad \theta = [\text{this} := e, x_i := l_i \mid i \in 1..n, y_i := l'_i \mid i \in 1..k, z_i := e_i \mid i \in 1..d] \quad P; E; s \vdash e_j : \theta(t_j) \ \& \ \bar{C}_j \quad \forall j \in 1..d \quad P; E \vdash \theta(t) \ \& \ \bar{C}' \quad \bar{C}'' = \bar{C} \cup \bar{C}_{1..d} \cup \bar{C}' \cup \{\theta(s') \subseteq s\}}{P; E; s \vdash e.\text{mn}(l'_{1..k})(e_{1..d}) : \theta(t) \ \& \ \bar{C}''}$$

$$\frac{P; E; s \vdash x : t' \ \& \ \bar{C} \quad P; E; s \cup \{x\} \vdash e : t \ \& \ \bar{C}'}{P; E; s \vdash \text{synchronized } x \ e : t \ \& \ (\bar{C} \cup \bar{C}')}$$

$$\frac{P; E; s \vdash e : \text{cn}(l_{1..n}) \ \& \ \bar{C} \quad \text{class } \text{cn}(\text{ghost } x_{1..n}) \ \{ \dots \text{meth} \dots \} \in P \quad \text{meth} = t' \ \text{run}(\text{ghost } \text{tl_lock})() \ \text{requires } \text{tl_lock} \ \{e'\}}{P; E; s \vdash e.\text{fork} : t \ \& \ C}$$